

Solution 1

November 5, 2019



Problem 1

Determine whether the following sentences are propositions. Try to determine the truth value if they are propositions. Otherwise, clarify the reason why they are not propositions.

- The Riemann Hypothesis is right.
- 12 is a prime number.
- $x + y = 6$
- What a beautiful flower!
- $3 - 1$
- Tom stays at home on weekends.
- This sentence is wrong.

Solution.

1. Yes. I don't know the truth value.
2. Yes. F.

3. No. x and y are variables.
4. No. It is not a statement.
5. No. It is not a statement.
6. Yes. T.
7. No. It's a paradox.

Problem 2

Use variables and connectives in propositional logic to formally represent the following statement.

- If the cheese is gone, either Tom eats it or Jerry eats it.
- Tom won't go home unless it rains.

Solution.

1. P denotes "the cheese is gone". Q denotes "Tom eats the cheese". R denotes "Jerry eats the cheese". $P \rightarrow (Q \vee R)$
2. P denotes "it rains", Q denotes "Tom will go home". $(\neg P) \rightarrow (\neg Q)$

Problem 3

P signifies "Tom is a cat" and Q signifies "Tom likes eating fish". Interpret the following proposition in natural language.

- $(\neg P) \wedge (\neg Q)$
- $P \rightarrow Q$
- $\neg(P \vee Q)$

Solution.

1. Tom is not a cat and Tom doesn't like eating fish.
2. If Tom is a cat, then Tom likes eating fish.
3. Tom is not a cat and Tom doesn't like eating fish.

Problem 4

Build the truth table of following propositions. You can merge some truth tables into one truth tables like p22 of slides. You will find some interesting things.

- $P \rightarrow Q, (\neg Q) \rightarrow (\neg P), (\neg P) \vee Q, (\neg P) \rightarrow (\neg Q)$
- $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

Solution.

P	Q	$P \rightarrow Q$	$(\neg Q) \rightarrow (\neg P)$	$(\neg P) \vee Q$	$(\neg P) \rightarrow (\neg Q)$
F	F	T	T	T	T
F	T	T	T	T	F
T	F	F	F	F	T
T	T	T	T	T	T

P	Q	R	$(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T