

Solution 2

November 6, 2019



Problem 1

Formalize the following statement with connectives and propositional variables. (Hint : p10 in the textbook)

Tom is a blue cat or Tom is a black cat.

Solution.

P denotes "Tom is a blue cat" and Q denotes "Tom is a black cat".

$(P \wedge (\neg Q)) \vee ((\neg P) \wedge Q)$.

Problem 2

If a connective connects n propositional variables, how many different connectives can we define? Given a well-formed formula with n propositional variables, how many rows if we build the truth table?

Solution.

2^{2^n} connectives. 2^n rows.

Problem 3

We define a new connective $\downarrow$:

$$P \downarrow Q = \neg(P \vee Q)$$

Please use only $\downarrow$ to represent $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$. You need to write the intermediate steps instead of only writing the final expression.

Solution.

$$\begin{aligned}\neg P &= \neg(P \vee P) \\ &= P \downarrow P\end{aligned}$$

$$\begin{aligned}P \wedge Q &= \neg((\neg P) \vee (\neg Q)) \\ &= (\neg P) \downarrow (\neg Q) \\ &= (P \downarrow P) \downarrow (Q \downarrow Q)\end{aligned}$$

$$\begin{aligned}P \vee Q &= \neg(\neg(P \vee Q)) \\ &= \neg(P \downarrow Q) \\ &= (P \downarrow Q) \downarrow (P \downarrow Q)\end{aligned}$$

$$\begin{aligned}P \rightarrow Q &= (\neg P) \vee Q \\ &= ((\neg P) \downarrow Q) \downarrow ((\neg P) \downarrow Q) \\ &= ((P \downarrow P) \downarrow Q) \downarrow ((P \downarrow P) \downarrow Q)\end{aligned}$$

$$\begin{aligned}P \leftrightarrow Q &= (P \rightarrow Q) \wedge (Q \rightarrow P) \\ &= \neg((\neg(P \rightarrow Q)) \vee (\neg(Q \rightarrow P))) \\ &= \neg((\neg(\neg P \vee Q)) \vee (\neg(\neg Q \vee P))) \\ &= \neg(((\neg P) \downarrow Q) \vee ((\neg Q) \downarrow P)) \\ &= ((\neg P) \downarrow Q) \downarrow ((\neg Q) \downarrow P) \\ &= ((P \downarrow P) \downarrow Q) \downarrow ((Q \downarrow Q) \downarrow P)\end{aligned}$$

Problem 4

Write the formula of A according to T rows. Write the formula of B according to F rows.

P	Q	A	B
T	T	F	T
T	F	T	T
F	T	T	F
F	F	T	F

Solution.

$$A = (P \wedge (\neg Q)) \vee ((\neg P) \wedge Q) \vee ((\neg P) \wedge (\neg Q))$$

$$B = (P \vee (\neg Q)) \wedge (P \vee Q)$$

Problem 5

Prove the following equations with laws. Don't use truth table.

- $(P \rightarrow Q) \wedge P = P \wedge Q$
- $\neg(P \leftrightarrow Q) = (P \wedge \neg Q) \vee (\neg P \wedge Q)$
- $(P \vee Q) \leftrightarrow (P \wedge Q) = (P \wedge Q) \vee (\neg P \wedge \neg Q)$

Solution.

1.

$$\begin{aligned}
& (P \rightarrow Q) \wedge P \\
&= ((\neg P) \vee Q) \wedge P \\
&= ((\neg P) \wedge P) \vee (Q \wedge P) \\
&= F \vee (P \wedge Q) \\
&= P \wedge Q
\end{aligned}$$

2.

$$\begin{aligned}
& \neg(P \leftrightarrow Q) \\
&= \neg((P \rightarrow Q) \wedge (Q \rightarrow P)) \\
&= \neg(((\neg P) \vee Q) \wedge ((\neg Q) \vee P)) \\
&= (P \wedge (\neg Q)) \vee ((\neg P) \wedge Q)
\end{aligned}$$

3.

$$\begin{aligned}
& (P \vee Q) \leftrightarrow (P \wedge Q) \\
& = ((P \vee Q) \wedge P \wedge Q) \vee (\neg(P \vee Q) \wedge \neg(P \wedge Q)) \\
& = (P \wedge Q) \vee (\neg(P \vee Q) \wedge \neg(P \wedge Q)) \\
& = (P \wedge Q) \vee ((\neg P) \wedge (\neg Q) \wedge ((\neg P) \vee (\neg Q))) \\
& = (P \wedge Q) \vee ((\neg P) \wedge (\neg Q))
\end{aligned}$$

Problem 6

Determine if $(P \leftrightarrow Q) \leftrightarrow ((P \wedge \neg Q) \vee (Q \wedge \neg P))$ is a tautology, a contradiction or a satisfiable formula. You should use two ways to solve the problem – building a truth table and using laws to convert the formula.

Solution.

It's a contradiction.

P	Q	$(P \leftrightarrow Q) \leftrightarrow ((P \wedge \neg Q) \vee (Q \wedge \neg P))$
F	F	F
F	T	F
T	F	F
T	T	F

$$\begin{aligned}
& (P \leftrightarrow Q) \leftrightarrow ((P \wedge \neg Q) \vee (Q \wedge \neg P)) \\
& = (P \leftrightarrow Q) \leftrightarrow (\neg(\neg(P \wedge \neg Q)) \wedge \neg(Q \wedge \neg P))) \\
& = (P \leftrightarrow Q) \leftrightarrow (\neg(((\neg P) \vee Q) \wedge ((\neg Q) \vee P))) \\
& = (P \leftrightarrow Q) \leftrightarrow (\neg((P \rightarrow Q) \wedge (Q \rightarrow P))) \\
& = (P \leftrightarrow Q) \leftrightarrow (\neg(P \leftrightarrow Q)) \\
& = F
\end{aligned}$$