Solution 7

November 21, 2019

Problem 1

Write preconditions and postconditions of the following incomplete Hoare triples to make them valid.

1. $\{a > 2\}a := a + 1\{\quad\}$
2. $\{\quad\}\ if(a == 0)\ then\ a := 1\ else\ a := 3\{a = 3\}$
3. $\{\quad\}\ a := b - c\ \{a = 5\}$
4. $\{a = 3\}while(a < 10)a := a + 1\{\quad\}$

Solution.

1. $\{a > 2\}a := a + 1\{a > 3\}$
2. $\{a = 4\}\ if(a == 0)\ then\ a := 1\ else\ a := 3\{a = 3\}$
3. $\{b - c = 5\}\ a := b - c\ \{a = 5\}$
4. $\{a = 3\}while(a < 10)a := a + 1\{a = 10\}$

Problem 2

Prove $\{a = v0 \wedge b = v1\}c := a; a := b; b := c\{a = v1 \wedge b = v0\}$.

Solution.

$(1)\{a = v1 \wedge c = v0\}b := c\{a = v1 \wedge b = v0\}$	AS
$(2)\{b = v1 \wedge c = v0\}a := b\{a = v1 \wedge c = v0\}$	AS
$(3)\{b = v1 \wedge a = v0\}c := a\{b = v1 \wedge c = v0\}$	AS
$(4)a = v0 \wedge b = v1 \rightarrow b = v1 \wedge a = v0$	*predicate logic*
$(5)\{a = v0 \wedge b = v1\}c := a\{b = v1 \wedge c = v0\}$	$SP(3)(4)$
$(6)\{a = v0 \wedge b = v1\}c := a; a := b\{a = v1 \wedge c = v0\}$	$SC(2)(5)$
$(7)\{a = v0 \wedge b = v1\}c := a; a := b; b := c\{a = v1 \wedge b = v0\}$	$SC(1)(6)$

Problem 3

Prove $\{T\}\ \ if(a == 1)\ then\ b := 3\ else\ b := 1\ \ \{b = 1 \vee b = 3\}$.

Solution.

$(1)\{3 = 1 \vee 3 = 3\}b := 3\{b = 1 \vee b = 3\}$	AS
$(2)\{1 = 1 \vee 1 = 3\}b := 1\{b = 1 \vee b = 3\}$	AS
$(3)T \wedge a = 1 \rightarrow 3 = 1 \vee 3 = 3$	*predicate logic*
$(4)\{T \wedge a = 1\}b := 3\{b = 1 \vee b = 3\}$	$SP(1)(3)$
$(5)T \wedge \neg(a = 1) \rightarrow 1 = 1 \vee 1 = 3$	*predicate logic*
$(6)\{T \wedge \neg(a = 1)\}b := 1\{b = 1 \vee b = 3\}$	$SP(2)(5)$
$(7)\{T\}\ \ if(a == 1)\ then\ b := 3\ else\ b := 1\ \ \{b = 1 \vee b = 3\}$	$CD(4)(6)$

Problem 4

Prove $\{a = 3\}\ \ while(a! = 10)\ a := a + 1\ \ \{a = 10\}$. The loop invariant is $a \leq 10$.

Solution.

$$
\begin{array}{lr}
(1)\{a+1 \leq 10\}a := a+1\{a \leq 10\} & AS \\
(2)a \leq 10 \wedge a \neq 10 \rightarrow a+1 \leq 10 & predicate\ logic \\
(3)\{a \leq 10 \wedge a \neq 10\}a := a+1\{a \leq 10\} & SP(1)(2) \\
(4)\{a \leq 10\}\ while(a! = 10)\ a := a+1\ \{a \leq 10 \wedge a = 10\} & WHP(3) \\
(5)a \leq 10 \wedge a = 10 \rightarrow a = 10 & predicate\ logic \\
(6)\{a \leq 10\}\ while(a! = 10)\ a := a+1\ \{a = 10\} & WC(4)(5) \\
(7)a = 3 \rightarrow a \leq 10 & predicate\ logic \\
(8)\{a = 3\}\ \ while(a! = 10)\ a := a+1\ \ \{a = 10\} & SP(6)(7)
\end{array}
$$

Problem 5

Write a weaker precondition and a stronger postcondition for $\{b = 7\}a := b+3\{a > 5\}$.

Solution.
weaker precondition: $b > 2$.
stronger postcondition: $a = 10$.

Problem 6

Compute $wlp(a := b-1;\ \ if(a == 10)\ then\ c := a+1\ else\ c := a+2 \quad , \quad c > 4)$

Solution.

$$
\begin{aligned}
&wlp(a := b-1;\ \ if(a == 10)\ then\ c := a+1\ else\ c := a+2 \quad , \quad c > 4) \\
=&wlp(a := b-1, (a = 10 \rightarrow a+1 > 4) \wedge (a \neq 10 \rightarrow a+2 > 4)) \\
=&(b-1 = 10 \rightarrow b-1+1 > 4) \wedge (b-1 \neq 10 \rightarrow b-1+2 > 4)
\end{aligned}
$$