

Discrete Mathematics

(for Computer Science)

Zhaoguo Wang

Why Choose CDM?



4 yrs in SJTU



For graduation



Graduate school



Interview



Startup

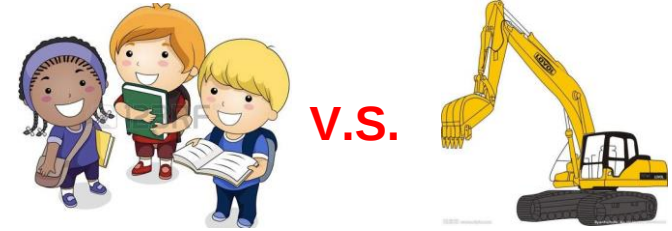


Programmer
/ Researcher



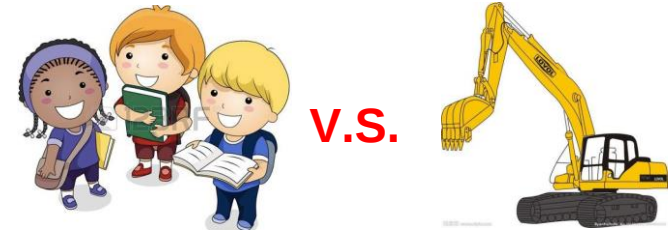
Why Choose CDM?

What makes you distinguished?



Why Choose CDM?

What makes you distinguished?



Understand “laws of physics” in CS

Be able to answer the fundamental questions.

Questions To Be Answered In CDM

What problems can you solve with a computer?

How can you be certain of your answer to these questions?

How Do These Two Questions Affect Each Path?



Interview

Interview Question:
what languages r u good at?



Java, Java Spring, Ruby, Python



Java, C++, .Net, C, Go, Rust



Besides above all, assembly language, Coq, TLA and so on...

How Do These Two Questions Affect Each Path?



Graduate school

Major concern by Profs.
Is the student a logical person?

He got **a grade of A** for CDM.

He can have logical thinking and answer the questions in a very logical way.

How Do These Two Questions Affect Each Path?



Startup

When talk to investors
The first feeling is important

His story makes sense.

He has a very clear mind.

How Do These Two Questions Affect Each Path?



Programmer
/ Researcher

**How do you know your
program is correct?**



There is no segmentation fault.



It passes all tests.



I can prove it.

How To Answer These Two Questions?

What problem can you solve with a computer?

How can you be certain of your answer to these questions?

大纲

- 图论

- 图的基本概念
- 道路与回路
- 树

- 逻辑

- 命题逻辑
- 谓词逻辑
- 公理系统

- 集合论

- 集合
- 关系
- 函数

大纲

- 图论

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- 函数

?

大纲

- 图论

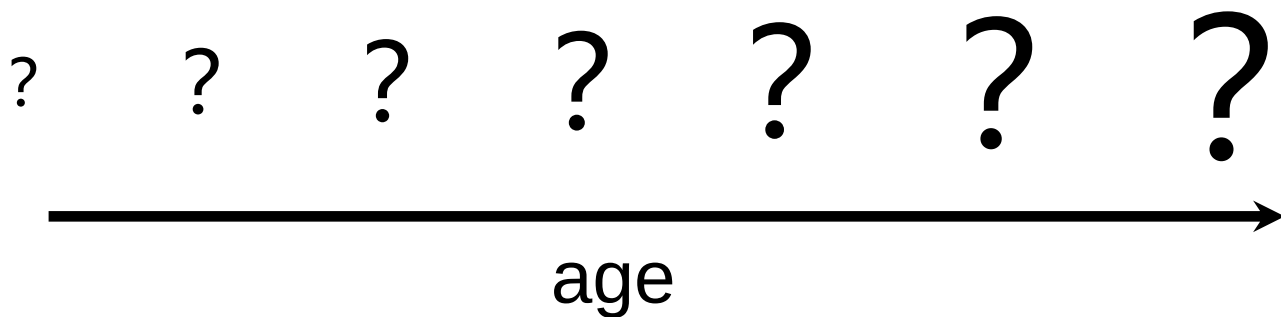
- 图的基本概念
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大纲

- 图论

- 图的基本概念

- 道路与回路

- 树

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这^将不^是一^门普^通的
离^散数^学

? ? ? ? ? ? ?
age



Be Practical than Attractive

Is it solvable?



```
bool is_power2(unsigned int v)
{
    unsigned int i;
    for(i = 0; i <= 31; ++i)
    {
        if(v == (1 << i))
            return true;
    }
    return false;
}
```



How to prove its correctness?



Three Parts of CDM

Part I. Reasoning. (8 ~ 9 *Weeks*)

How to prove the correctness?

Part II. Computability. (~ 6 *Weeks*)

Is the problem computable (solvable)?

Part III. Probability. (1 ~ 1.5 *Weeks*)

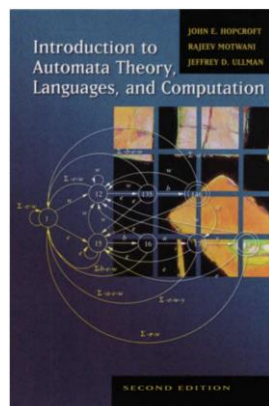
How does computer solve continuous problem?
(Underneath the ML)

References

Textbooks



《数理逻辑与集合论》第2版
• 石纯一 著，清华大学出版社



John E. Hopcroft, Rajeev Motwani and Jeffrey D. Ullman,
Introduction to Automata Theory, Languages, and
Computation, Pearson, 2001

References

- Courses**
- **Stanford CS103**, <https://web.stanford.edu/class/cs103>
 - **NYU G22.2390-001**, <https://cvc4.cs.stanford.edu/logic/>
 - CMU CDM <http://www.cs.cmu.edu/~cdm/>
 - UB CS70 <http://inst.eecs.berkeley.edu/~cs70/fa16/>
 - CMU-CS122 <https://www.cs.cmu.edu/~iliano/courses/17F-CMU-CS122/>
 - **Stanford CS103**, <https://web.stanford.edu/class/cs103>
 - CMU 15-453 <https://www.cs.cmu.edu/~fp/courses/flac/>
 - Columbia, COMS W3261, <http://www.cs.columbia.edu/~aho/cs3261/>
 - IIT Jodhpur, CS222, <http://krchowdhary.com/toc/cs222.html>
 - 南京大学离散数学课程

Policies

You must work alone on all assignments

- You may post questions on Piazza,
- You are encouraged to answer others' questions, but refrain from explicitly giving away solutions.

Labs & Exercises

- Assignments due at 11:59pm on the due date
- Everybody has 5 grace days
- Zero score after the due

Integrity and Collaboration Policy

We will enforce the policy strictly.

1. The work that you turn in must be yours
2. You must acknowledge your influences
3. You must not look at, or use, solutions from prior years or the Web, or seek assistance from the Internet
4. You must take reasonable steps to protect your work
 - You must not publish your solutions
5. If there are inexplicable discrepancies between exam and lab performance, we will over-weight the exam and interview you.

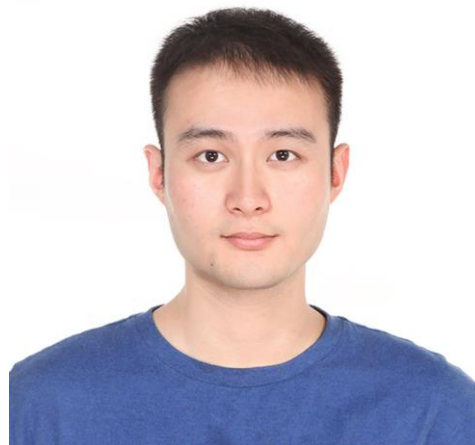
Faculty Information



Lecturer

Zhaoguo Wang (王肇国)

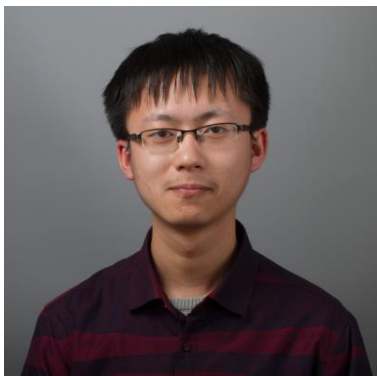
zhaoguowang@sjtu.edu.cn



Lecturer

Zeyu Mi (糜泽羽)

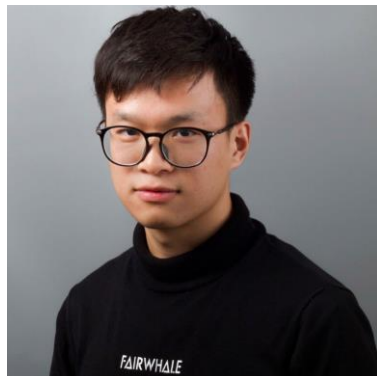
yzmizeyu@gmail.com



TA

Haoran Ding (丁浩然)

nhaorand@sjtu.edu.cn



TA

Yanfei Yang (杨雁飞)

yyf_sjtu@sjtu.edu.cn



TA

Chuzhe Tang (唐楚哲)

t.chuzhe@gmail.com

Class Info

Website:

<http://ipads.se.sjtu.edu.cn/courses/cdm>

Piazza signup:

<http://piazza.com/sjtu.org/fall2019/cdm>

Three Parts of CDM

Part I. Reasoning.

Part II. Computability.

Part III. Probability.

Part I: Reasoning

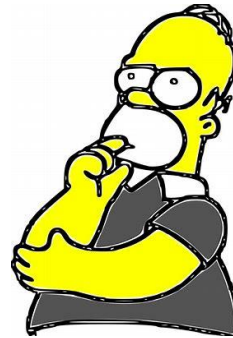
```
bool is_power2(unsigned int v)
{
    unsigned int i;
    for(i = 0; i <= 31; ++i)
    {
        if(v == (1 << i))
            return true;
    }
    return false;
}
```

Q: What is the semantic of this piece of code?

Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    unsigned int i;
    for(i = 0; i <= 31; ++i)
    {
        if(v == (1 << i))
            return true;
    }
    return false;
}
```

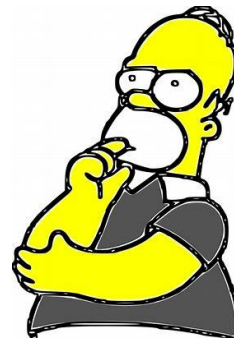
Hmmm... This looks correct, but tedious.



Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = !(v & (v - 1));
    return f;
}
```

Hmmm... This looks
clever, but suspicious.



Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = !(v & (v - 1));
    return f;
}
```

*Q: How to ensure the
correctness?*

Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = !(v & (v - 1));
    return f;
}
```

*Q: How to ensure the
correctness?*

Run some test cases?

Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = !(v & (v - 1));
    return f;
}
```

What if v is 0?

Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = v & !(v & (v - 1));
    return f;
}
```

Fix the bug.



Hmmm... Testing can find some bugs, but can not prove its correctness.



Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = v & !(v & (v - 1));
    return f;
}
```

Can we prove it?



Hmmm... Let me have a try with a pen. Maybe I can prove it with natural language.

Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = v & !(v & (v - 1));
    return f;
}
```

Can we prove it?

A Manual Proof



V is a 32-bits integer. V is a power of 2 iff there is one and only one bit is 1.

In “ $v \&\& !(v \& (v-1))$ ”, v ensures there is one 1-bit and $!(v \& (v-1))$ ensures there is only one 1-bit.



Part I: Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = v & !(v & (v - 1));
    return f;
}
```

*Can you ensure your logic
and every sentence is
correct?*



Hmmmm... Kill me...

Part I: Computer Aided Reasoning

```
bool is_power2(unsigned int v)
{
    bool f;
    f = v & !(v & (v - 1));
    return f;
}
```

*Ask the computer to prove
its correctness...
what should we do first?*

Part I: Computer Aided Reasoning

```
bool xor(unsigned int v)
{
    return v ^ v;
}
```


$$\begin{aligned} &((v_1 \wedge v_1) \vee ((\neg v_1) \wedge (\neg v_1))) \wedge \\ &((v_2 \wedge v_2) \vee ((\neg v_2) \wedge (\neg v_2))) \wedge \\ &((v_3 \wedge v_3) \vee ((\neg v_3) \wedge (\neg v_3))) \wedge \\ &\dots \\ &((v_{32} \wedge v_{32}) \vee ((\neg v_{32}) \wedge (\neg v_{32}))) \end{aligned}$$

A Propositional Formula

1.1 Propositional Logic

Step 1. Convert it into mathematical formula.

Step 2. Ask the computer to solve the formula.

Part I: Computer Aided Reasoning

```
void swap(int a, int b)
{
    a ^= b;
    b ^= a;
    a ^= b;
}
```


$$\begin{aligned} & (a1 \leftrightarrow (((((a1 \wedge (\neg b1)) \vee ((\neg a1) \wedge b1))) \wedge (\neg b1)) \vee \\ & ((\neg(((a1 \wedge (\neg b1)) \vee ((\neg a1) \wedge b1)))) \wedge b1))) \wedge \\ & (b1 \leftrightarrow (((((a1 \wedge (\neg b1)) \vee ((\neg a1) \wedge b1))) \wedge \\ & (\neg((((((a1 \wedge (\neg b1)) \vee ((\neg a1) \wedge b1))) \wedge (\neg b1)) \vee \\ & ((\neg(((a1 \wedge (\neg b1)) \vee ((\neg a1) \wedge b1)))) \wedge b1)))))) \vee \\ & ((\neg(((a1 \wedge (\neg b1)) \vee ((\neg a1) \wedge b1)))) \wedge \\ & (((((((a1 \wedge (\neg b1)) \vee ((\neg a1) \wedge b1))) \wedge (\neg b1)) \vee \\ & ((\neg(((a1 \wedge (\neg b1)) \vee ((\neg a1) \wedge b1)))) \wedge b1)))))) \wedge \\ & \dots \end{aligned}$$

A Propositional Formula

1.1 Propositional Logic

Step 1. Convert it into mathematical formula.

Step 2. Ask the computer to solve the formula.



It is complicated for both human being and computer.

Part I: Computer Aided Reasoning

```
void swap(int a, int b)
{
    a ^= b;
    b ^= a;
    a ^= b;
}
```

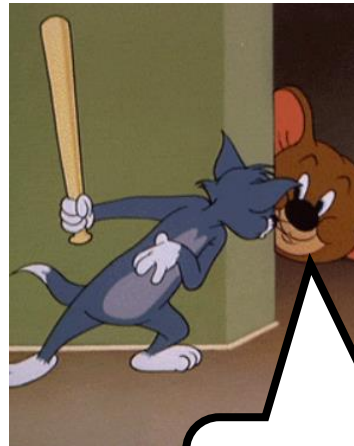

$$a = \text{XOR}(b, \text{XOR}(a, b)) \wedge$$
$$b = \text{XOR}(\text{XOR}(a, b), \text{XOR}(b, \text{XOR}(a, b)))$$

A Predicate Formula

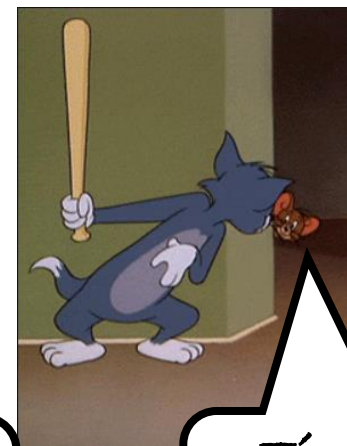
1.1 Propositional Logic

Step 1. Convert it into mathematical formula.

Step 2. Ask the computer to solve the formula.



propositional
logic



First-order
logic

Part I: Computer Aided Reasoning

```
void swap(int a, int b)
{
    a ^= b;
    b ^= a;
    a ^= b;
}
```


$$a = XOR(b, XOR(a, b)) \wedge$$
$$b = XOR(XOR(a, b), XOR(b, XOR(a, b)))$$

A Predicate Formula

1.1 Propositional Logic

Step 1. Convert it into mathematical formula.

Step 2. Ask the computer to solve the formula.

1.2 First Order Logic

Step 1. Convert it into first logic formula.

Step 2. Ask the computer to solve the formula.

Part I: Computer Aided Reasoning

```
int clone(unsigned int n)
{
    int i = 0;
    while(i < n)
    {
        i = i + 1;
    }
    return i;
}
```

*Loop can not be addressed
by first-order logic.*

*Can we tell computer
this is a loop?*

1.1 Propositional Logic

Step 1. Convert it into
mathematical formula.

Step 2. Ask the computer
to solve the formula.

1.2 First Order Logic

Step 1. Convert it into
first logic formula.

Step 2. Ask the computer
to solve the formula.



Part I: Computer Aided Reasoning

```
int clone(unsigned int n)
{
    int i = 0;
    while(i < n)
    {
        i = i + 1;
    }
    return i;
}
```



```
method clone(n: nat) returns (b : nat)
    ensures b == n
{
    var i := 0;
    while i < n
        invariant 0 <= i
    {
        i := i + 1;
    }
    return i;
}
```

1.1 Propositional Logic

Step 1. Convert it into mathematical formula.

Step 2. Ask the computer to solve the formula.

1.2 First Order Logic

Step 1. Convert it into first logic formula.

Step 2. Ask the computer to solve the formula.

1.3 Interactive Proof

Step 1. Axiom system

Step 2. Ask the computer to check the invariants

Part I: Computer Aided Reasoning

```
int clone(unsigned int n)
{
    int i = 0;
    while(i < n)
    {
        i = i + 1;
    }
    return i;
}
```



```
method clone(n: nat) returns (b : nat)
  ensures b == n
{
    var i := 0;
    while i < n
      invariant 0 <= i
    {
        i := i + 1;
    }
    return i;
}
```

		Description	Line	Column
✖	1	A postcondition might not hold on this return path.	10	3
	2	This is the postcondition that might not hold.	2	12

Part I: Computer Aided Reasoning

```
int clone(unsigned int n)
{
    int i = 0;
    while(i < n)
    {
        i = i + 1;
    }
    return i;
}
```



```
method clone(n: nat) returns (b : nat)
    ensures b == n
{
    var i := 0;
    while i < n
        invariant 0 <= i && i <= n
    {
        i := i + 1;
    }
    return i;
}
```

Dafny 2.3.0.10506

Dafny program verifier finished with 1 verified, 0 errors
Program compiled successfully

Part I: Computer Aided Reasoning Overview

1.1 Propositional Logic

Step 0. Proof.

Step 1. Convert program into mathematical formula.

Step 2. Ask the computer to solve the formula.

1.2 First Order Logic

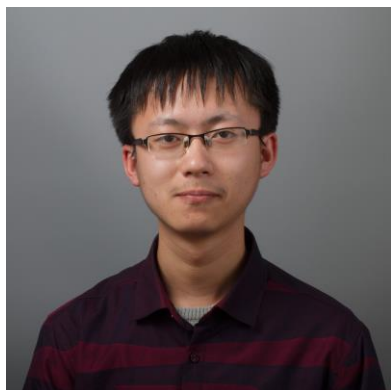
Step 1. Convert program into first logic formula.

Step 2. Ask the computer to solve the formula.

1.3 Interactive Proof

Step 1. Axiom system

Step 2. Ask the computer to check the invariants



3 Labs in total.
Good luck to all!

TA

Haoran Ding (丁浩然)

nhaorand@sjtu.edu.cn