

# Linearizability

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Some slides are adapted from Prof. Jinyang Li's DS class

# CSE – Distributed System Part

Part I: Fault Tolerance

Part II: Consistency

- Linearizability
- Paxos

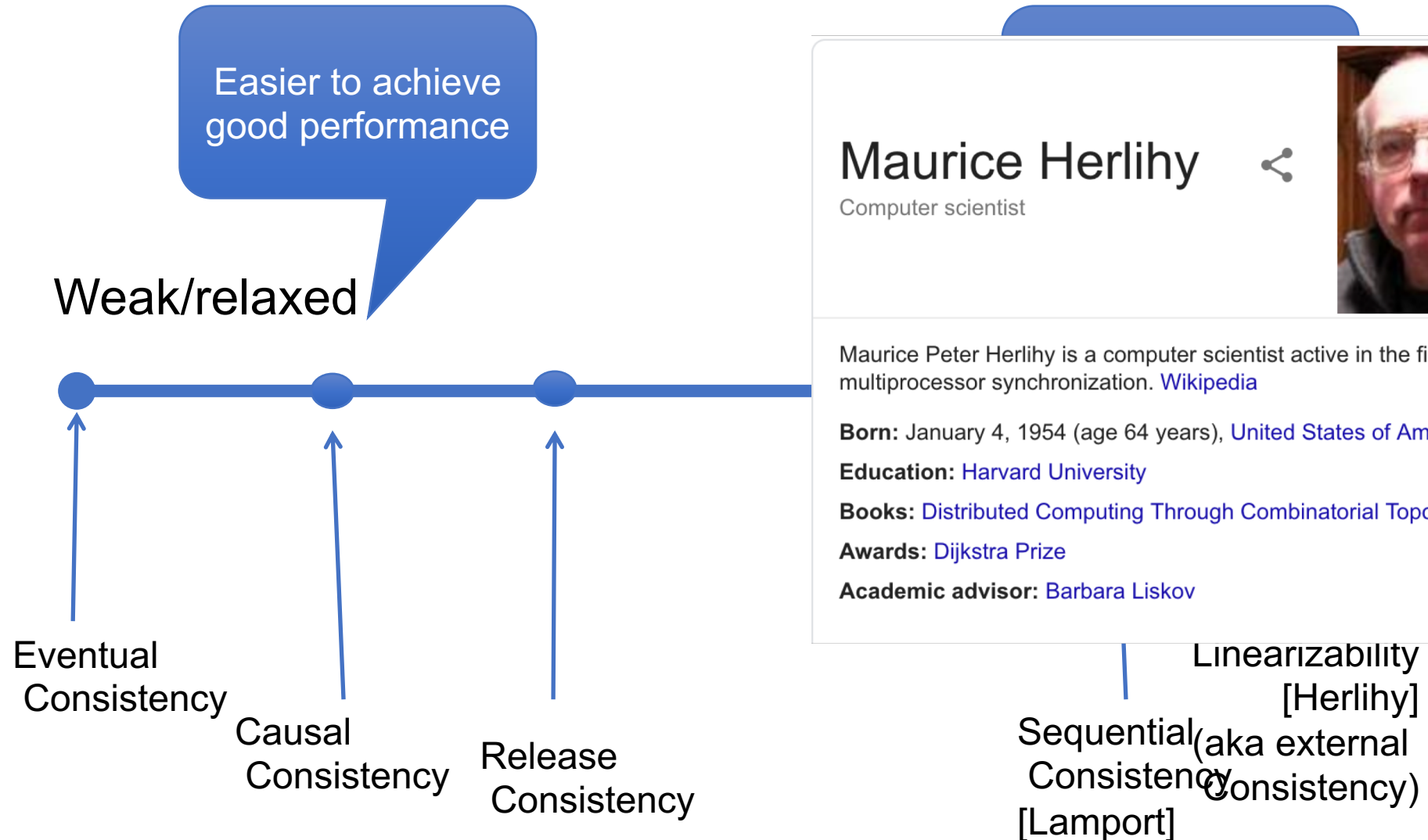
Part III: Distributed Transaction

# Consistency model

Consistency model = meaning of operations in the face of concurrency and failure

- A contract between system designers and application programmers
- A set of rules dictating what allowed system behaviors are

# Spectrum of Consistency Models



Maurice Herlihy

Computer scientist



Maurice Peter Herlihy is a computer scientist active in the field of multiprocessor synchronization. [Wikipedia](#)

**Born:** January 4, 1954 (age 64 years), [United States of America](#)

**Education:** [Harvard University](#)

**Books:** [Distributed Computing Through Combinatorial Topology](#)

**Awards:** [Dijkstra Prize](#)

**Academic advisor:** [Barbara Liskov](#)

# Consistency models are abstract

- Independent from any implementation
- Why specifying an abstract consistency model?
  - Application programmers do not have to understand the system implementation
  - System developers can reason correctness of implementation & optimizations
    - add caching
    - add replication
    - shard data across machines etc.

# Application programmers' expectation

Thread-1

(running on client machine 1)

```
Put("x", 1);  
Put("y", 1);
```

Thread-2

(running on client machine 2)

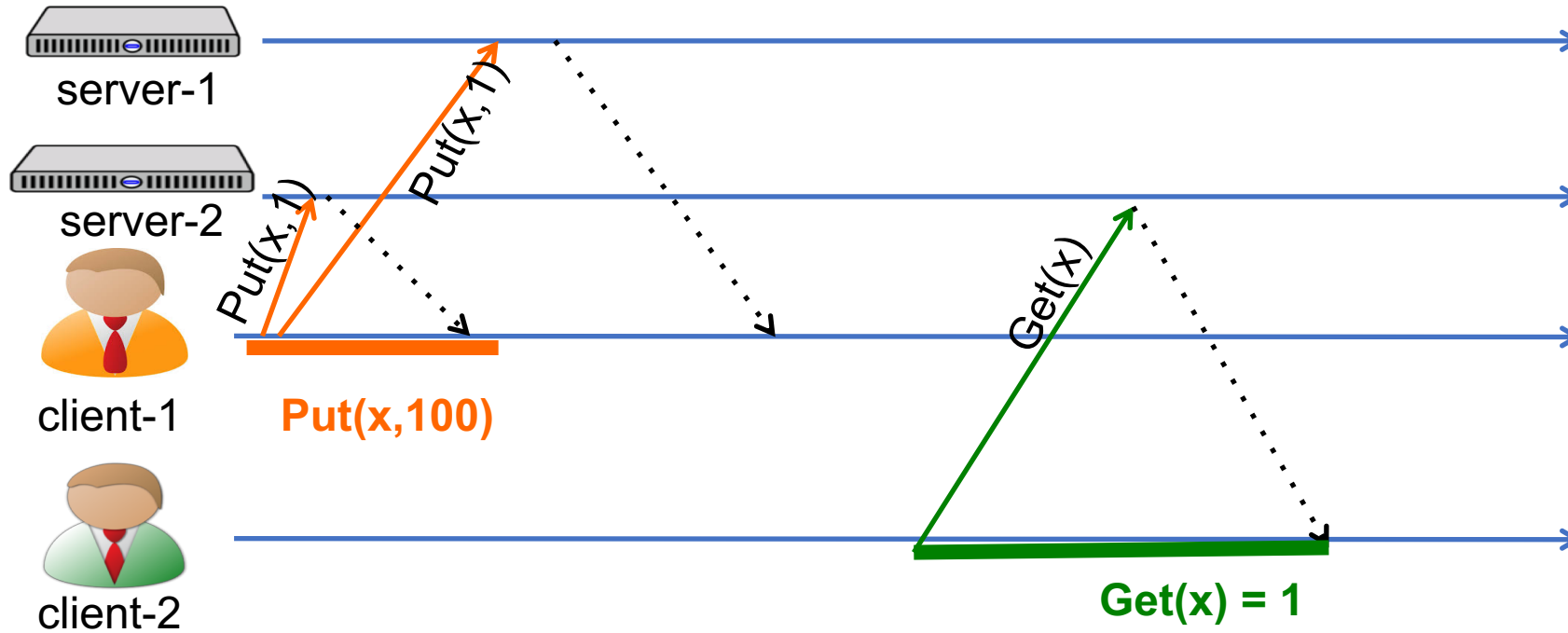
```
Get("Y")=?;  
Get("X")=?;
```

## What are expected values for Gets?

- Put(x,1), Put(y,1), Get(y)=1, Get(x)=1
- Put(x,1), Get(y)=0, Put(y,1), Get(x)=1
- ...
- Get(y)=0, Get(x)=0, Put(x,1), Put(y,1)

- 6 potential interleavings:
- 3 potential readings: (x=0,y=0) or (x=1 y=1) or (x=1 y=0)

# A strawman key-value store

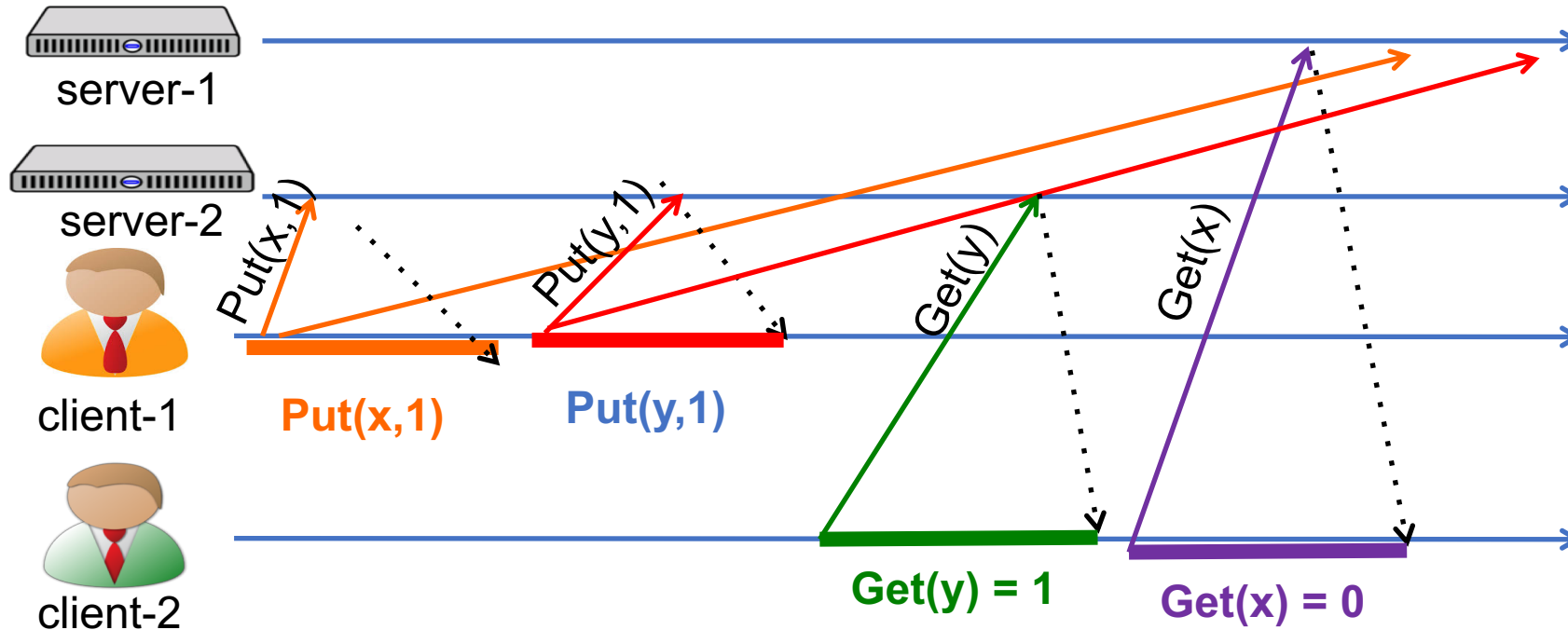


- System implementation:
  - Clients send writes to both replicas in parallel, wait for the fastest reply
  - Clients read from either replica

# Does strawman satisfies programmers' expectation?

- Programmers' expectation: 3 read values
  - $(x=0, y=0)$ , or  $(x=1, y=1)$ , or  $(x=1, y=0)$
- Is the expectation fulfilled w/ a single server replica?
- Is the expectation fulfilled w/ 2 replicas?
  - Is  $x=0, y=1$  possible under strawman?

# A strawman key-value store



- System implementation:
  - Clients send writes to both replicas in parallel, wait for the fastest reply
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# How to define a strong consistency model?

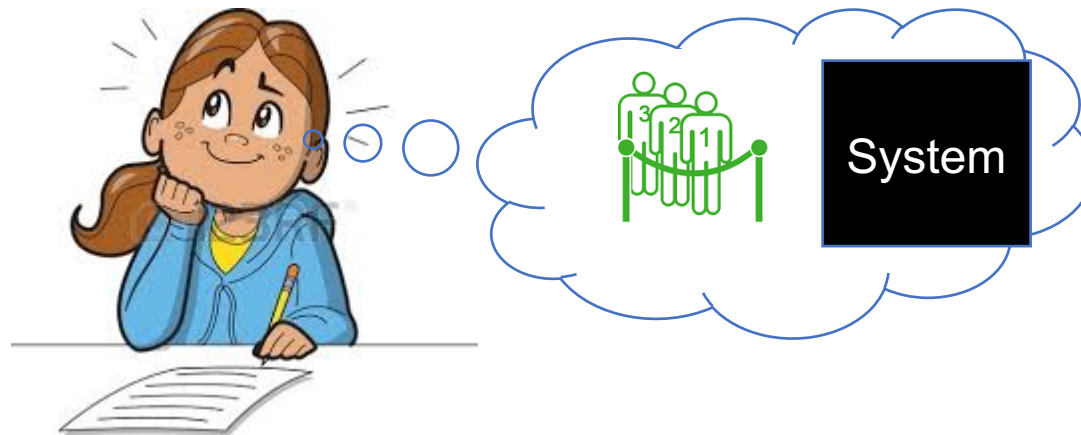
## Reality

- Concurrent execution
- replicated data



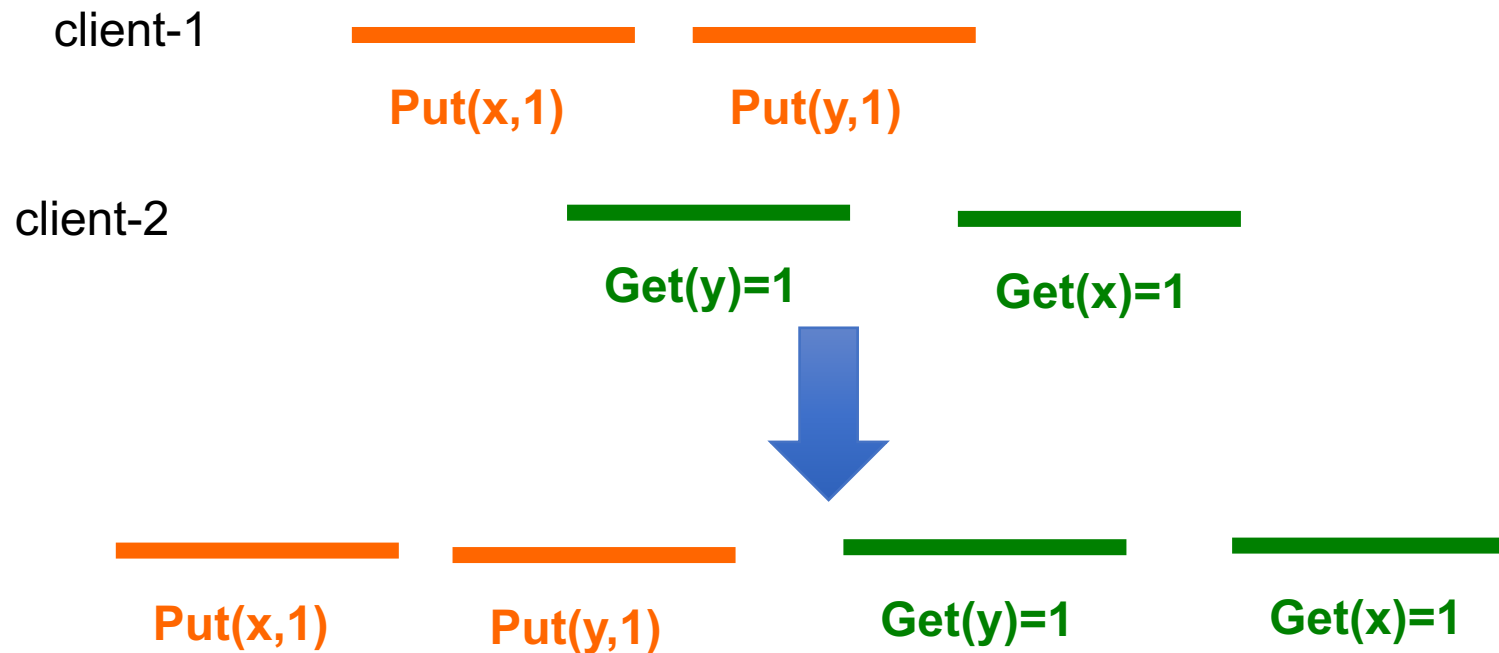
## Simple abstract mental model

- “Sequential” behavior
- “One-copy” of data



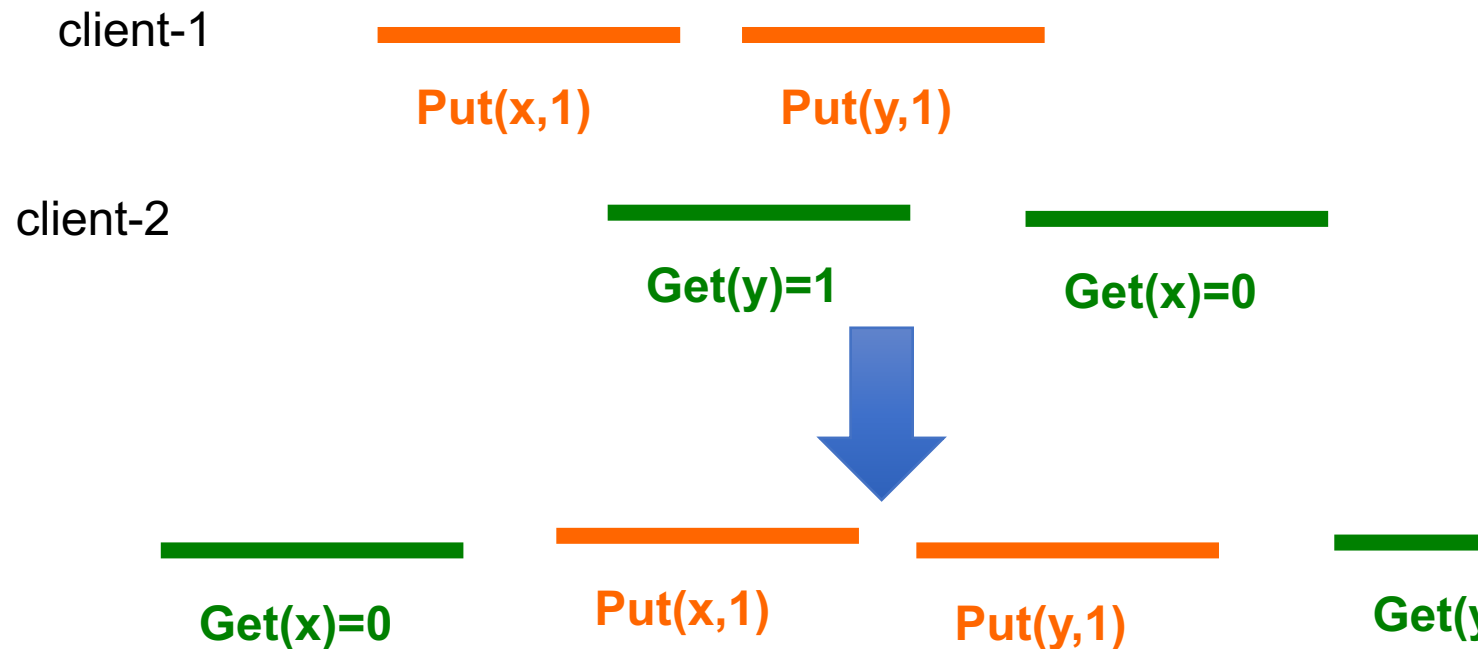
# How to define a strong consistency model?

Reduce a concurrent history to an equivalent serial execution history



# How to define a strong consistency model?

- Not all equivalent serial execution history “make sense”

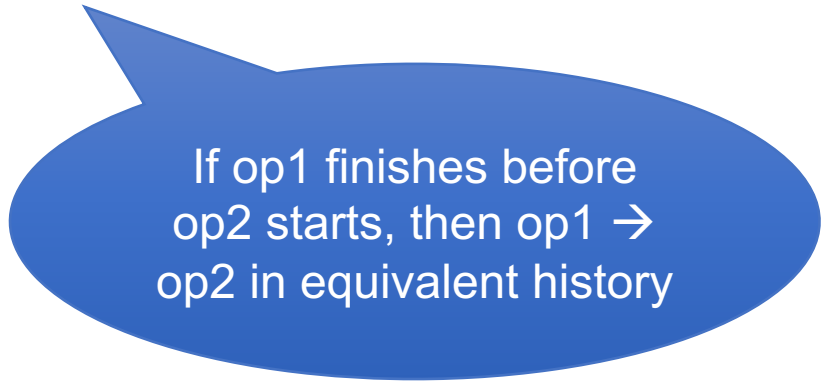


# What are allowed equivalent serial history?

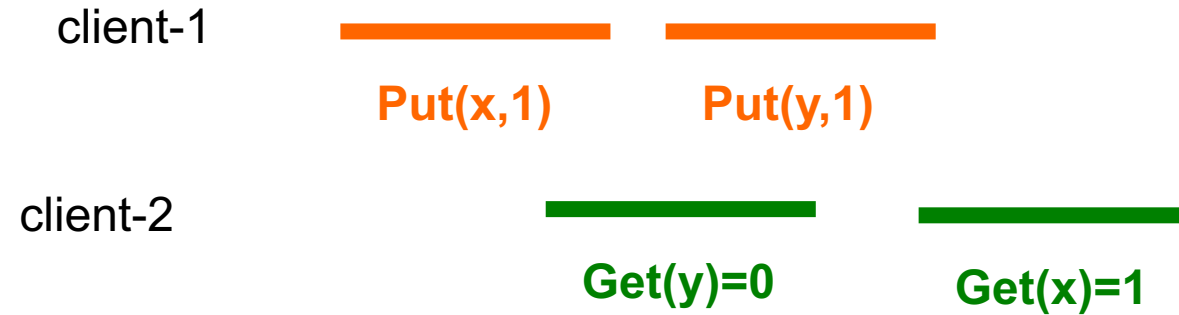
Certain orders in the original execution should be preserved in the equivalent history

What are possibilities?

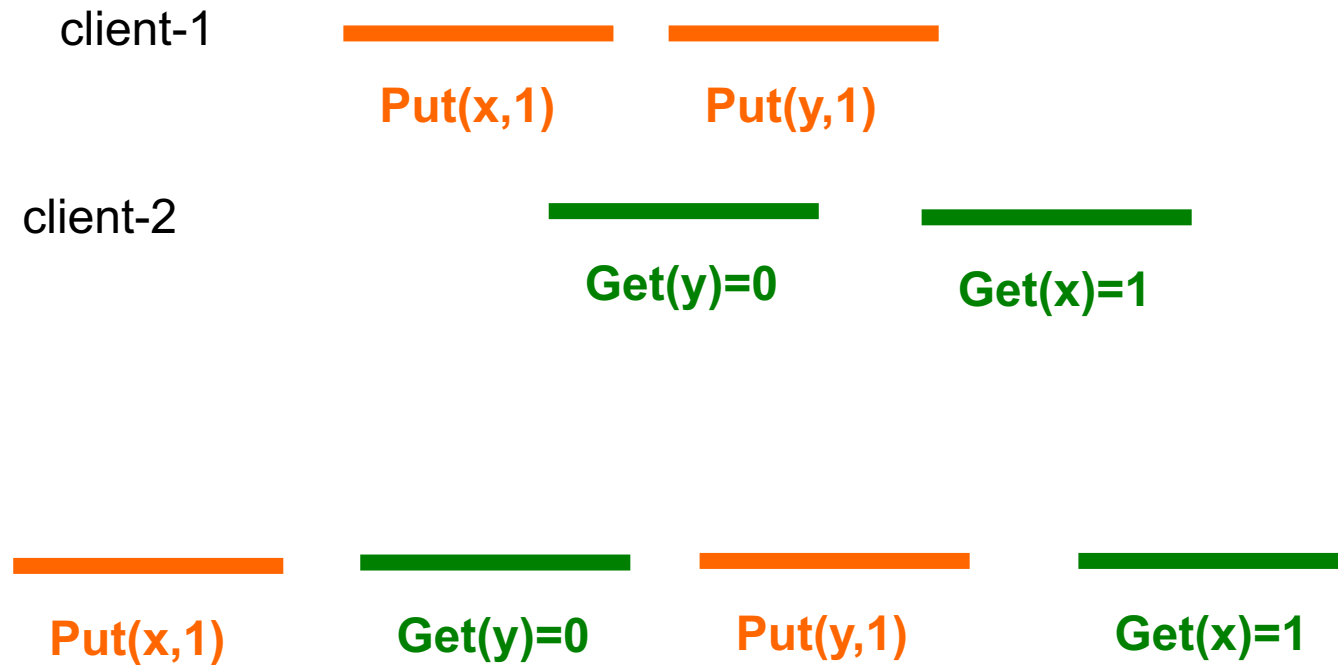
1. Global op issue order
2. Global op completion order
3. per-thread issue (and completion) order
4. global “completion-to-issue” order



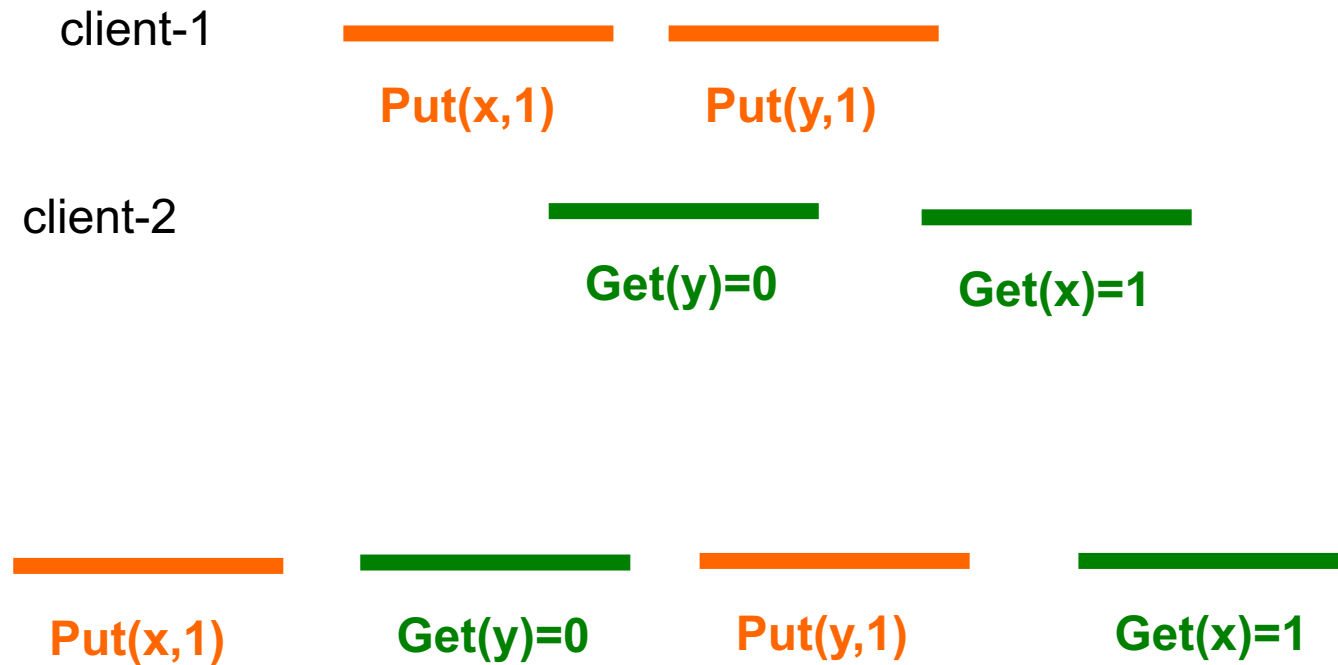
If op1 finishes before  
op2 starts, then op1 →  
op2 in equivalent history



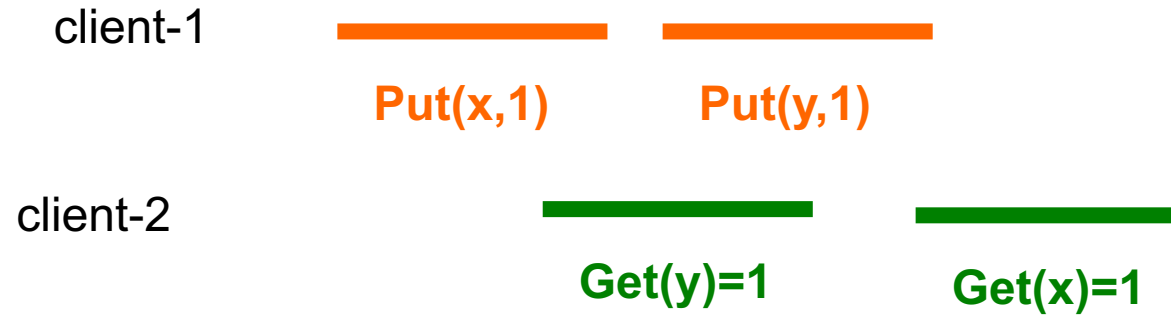
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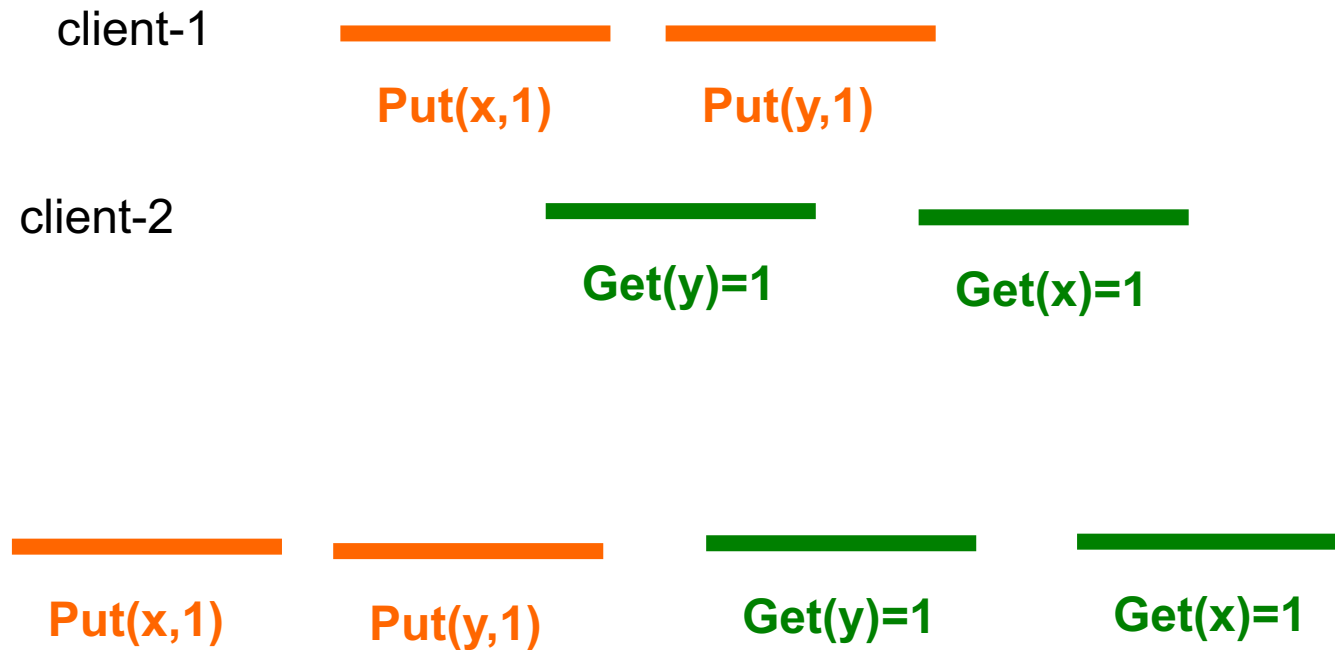
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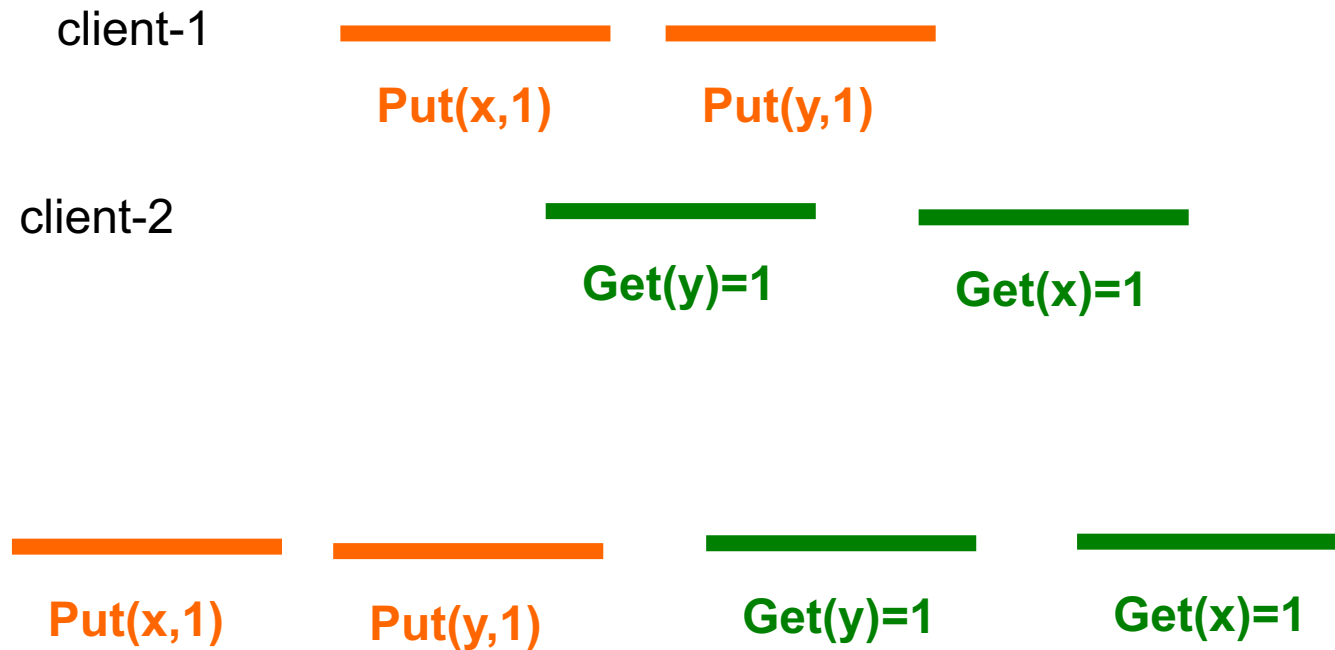
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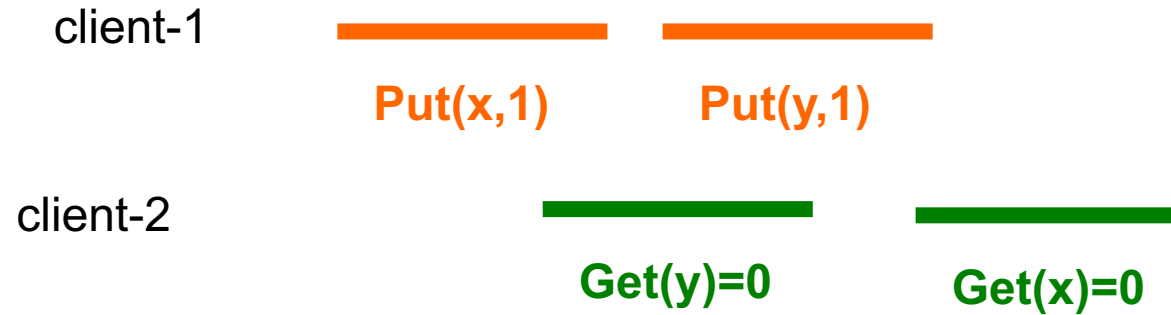
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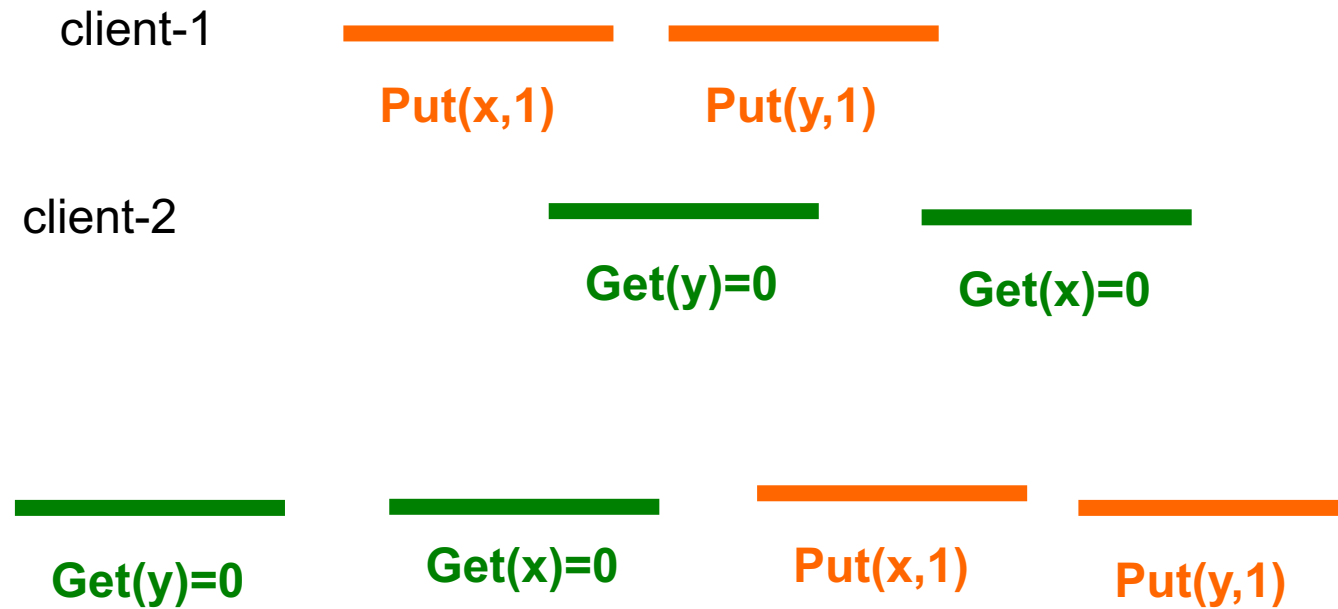
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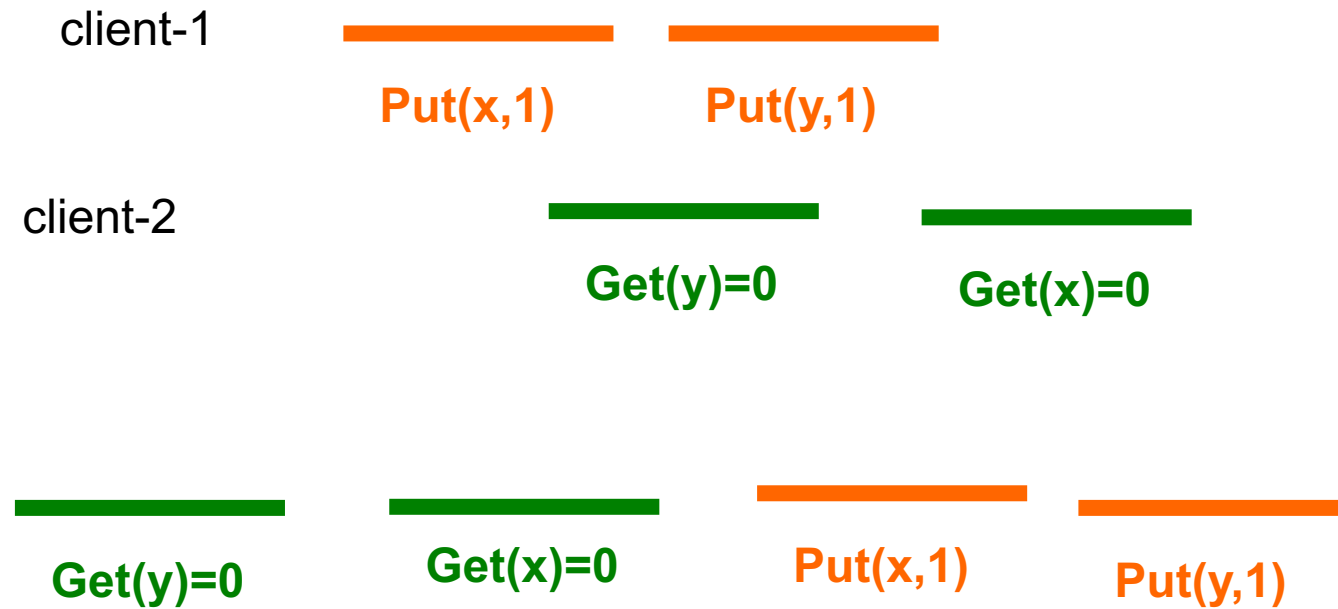
- X 1. Global issue order
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- 4. Global completion-to-issue order



1. Global issue order
2. Global completion order
3. Per-thread issue (and completion) order
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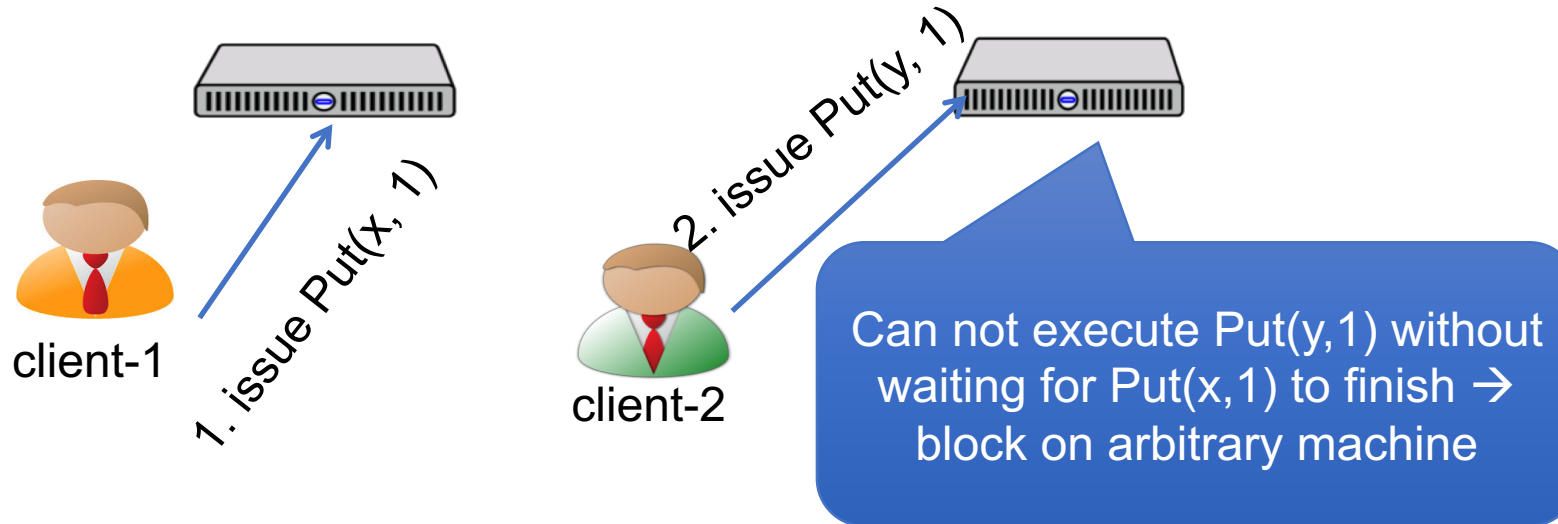
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- X 1. Global issue order
- X 2. Global completion order
- 3. Per-thread issue (and completion) order
- X 4. Global completion-to-issue order

# Strong consistency models

- Preserving global issue/completion order is impractical
  - → extreme blocking behavior



# Strong consistency models

- Sequential consistency
  - equivalent serial history must obey per-process issue/completion order
- Linearizability
  - equivalent serial history must obey global “completion-to-issue” order
- Which one is stronger?

1. Global issue order
2. Global completion order



stronger

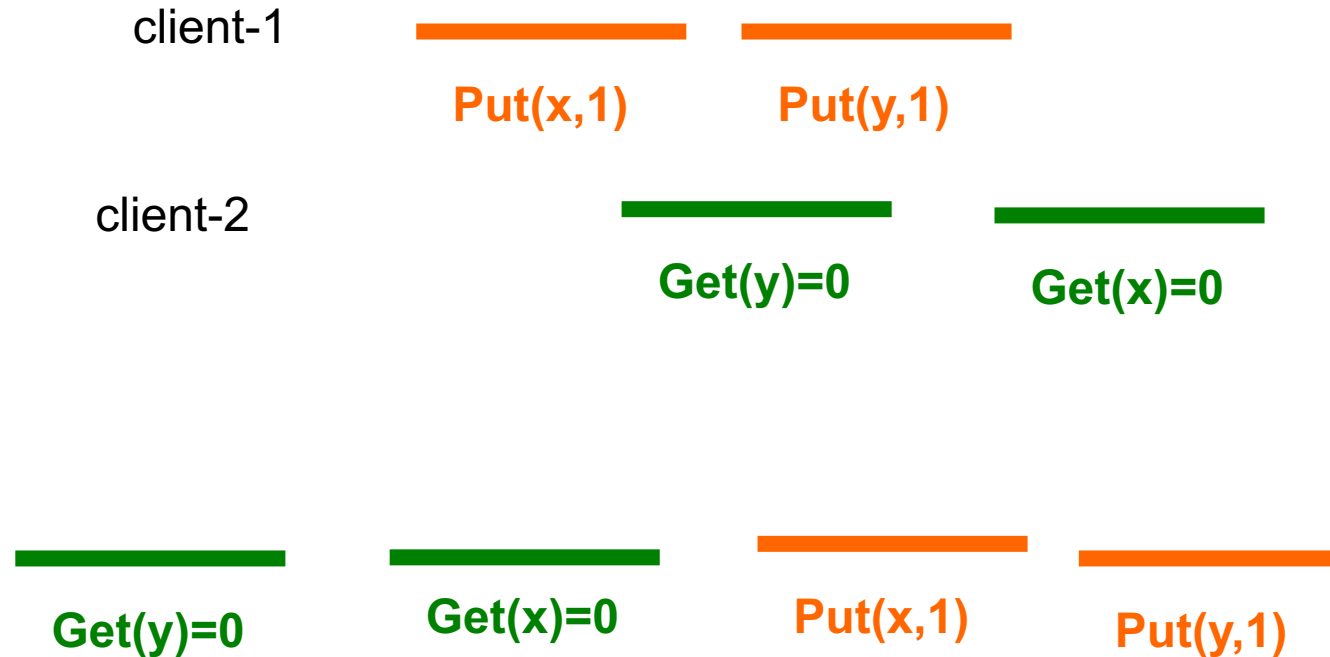
4. Global completion-to-issue order (Linearizability)



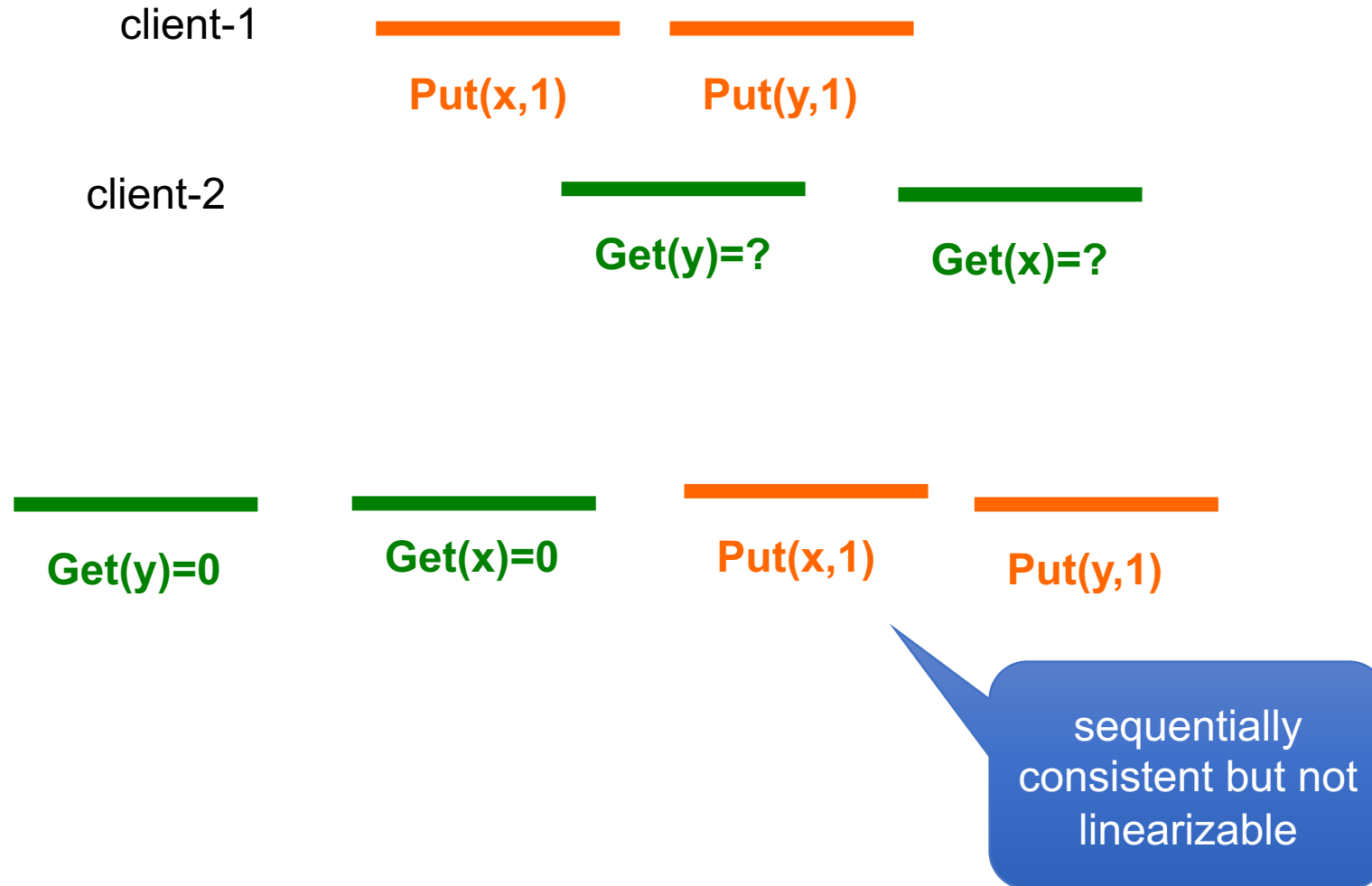
stronger

3. Per-thread issue (and completion) order (sequential consistency)

# Sequential consistency vs. linearizability

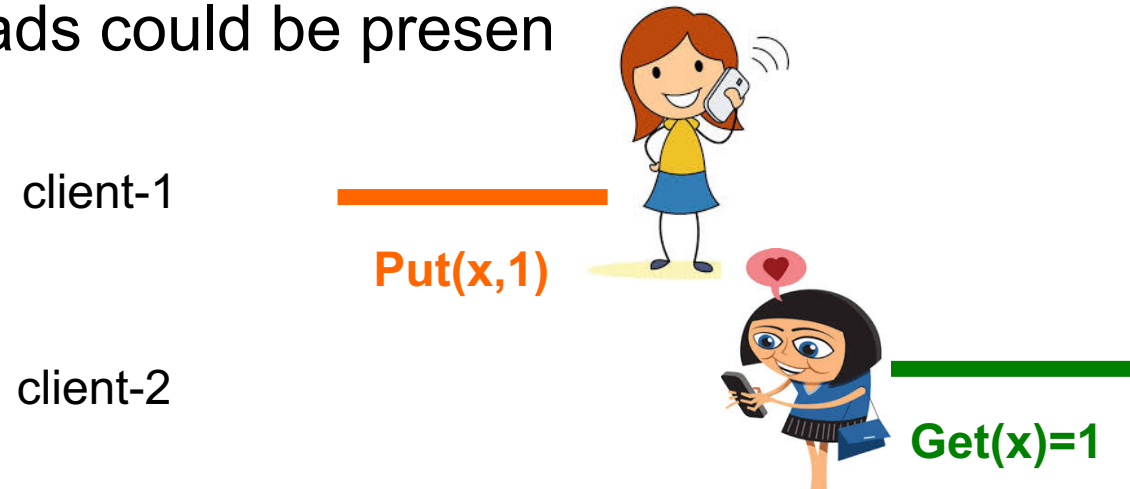


# Sequential consistency vs. linearizability



# Sequential consistency vs. linearizability

- Does the difference between the two matter?
  - Matters only when external communication between threads could be present

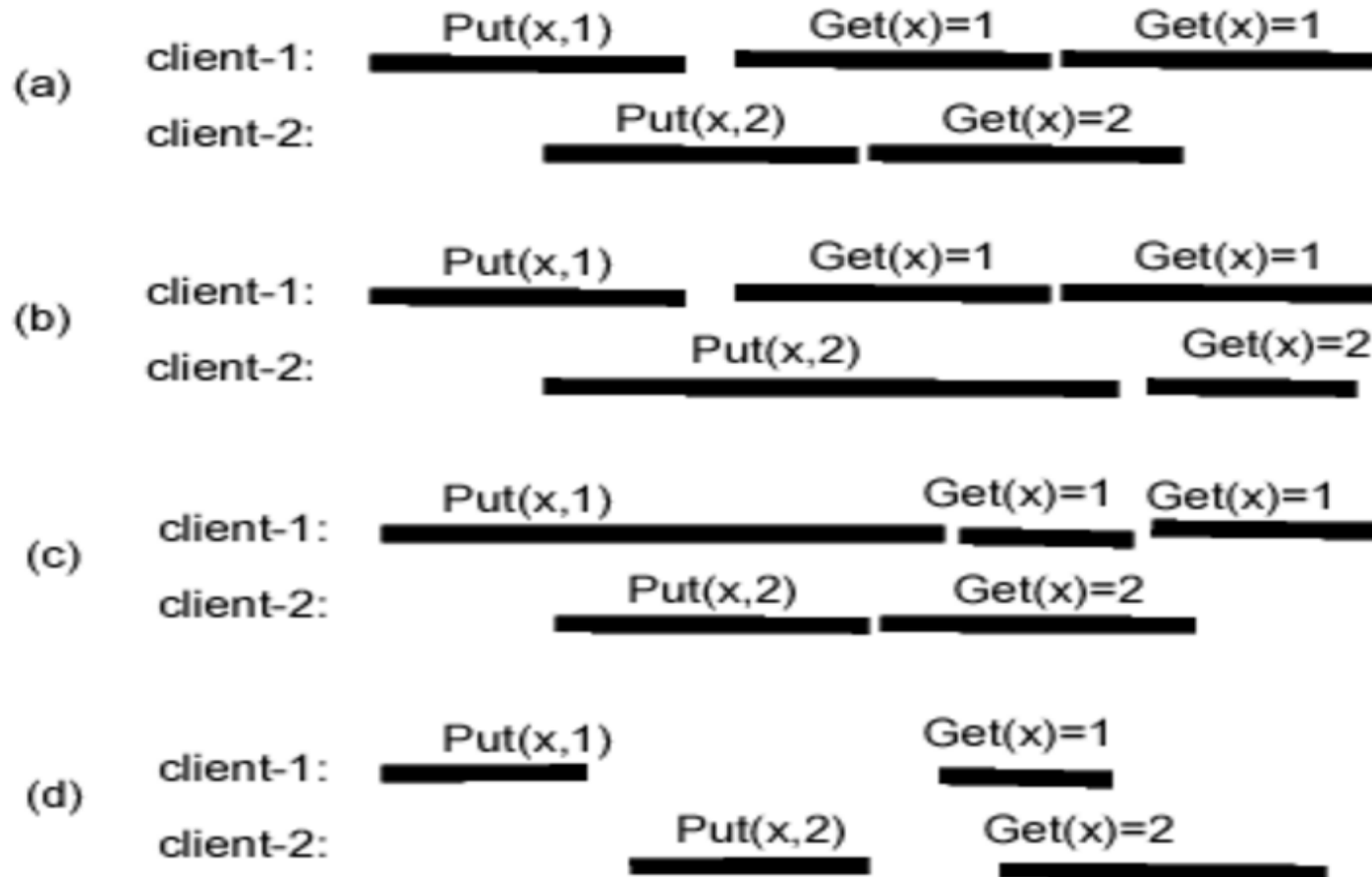


- Both are equally good w/o external communication
  - e.g. Multi-threaded programs sharing sequential consistent global memory

# Why linearizability is a good strong model?

- It is strong
  - Strongest possible practical model
- It allows scalable system design
  - (the local property) if each object is linearizable, then the whole system is linearizable
  - Scale system → shard objects across servers

# Recap exercise: which of the following histories are linearizable?



# What are the implementations of linearizability

- Case-study: key-value store
  - Single server
  - N servers: server-i stores keys,  $k \bmod N = i$
  - 3 servers replicating data using viewstamp replication



VR log contains  
both read and  
write ops

# What we've learnt until now?

## Strong consistency model: linearizability

- execution is equivalent to a serial history that preserves global completion-to-issue order

## How to implement linearizability

- no replication
- viewstamp replication

# Consensus – Consistency Replication

Allow a set of nodes to agree on something in the presence of a number of failure.

- Replica servers agree on the same sequence of operations for execution
- Linearizable System (e.g., VR)

# Paxos Solves The Consensus Problem



The screenshot shows the ACM A.M. Turing Award website. The header features the ACM logo, the text "A.M. TURING AWARD", and a search bar. Below the header is a grid of 24 small portraits of past winners. The main content area is titled "A.M. TURING AWARD WINNERS BY..." and has three tabs: "ALPHABETICAL LISTING", "YEAR OF THE AWARD", and "RESEARCH SUBJECT". The "ALPHABETICAL LISTING" tab is selected, showing a large portrait of Leslie Lamport on the left. To the right of the portrait, the text "LESLIE LAMPORT" is displayed with a green "DL" badge. Below the name, it says "United States – 2013". Under the heading "CITATION", the text reads: "For fundamental contributions to the theory and practice of distributed and concurrent systems, notably the invention of concepts such as causality and logical clocks, safety and liveness, replicated state machines, and sequential consistency."

acm

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ALPHABETICAL LISTING YEAR OF THE AWARD RESEARCH SUBJECT



**LESLIE LAMPORT** DL

United States – 2013

**CITATION**

For fundamental contributions to the theory and practice of distributed and concurrent systems, notably the invention of concepts such as causality and logical clocks, safety and liveness, replicated state machines, and sequential consistency.

Correctness: servers execute the same sequence of log



No two different ops are committed at the same log position



All committed ops in view  $V - 1$  must be known to primary in view  $V$

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Single Decree Consensus

# Single Decree Consensus

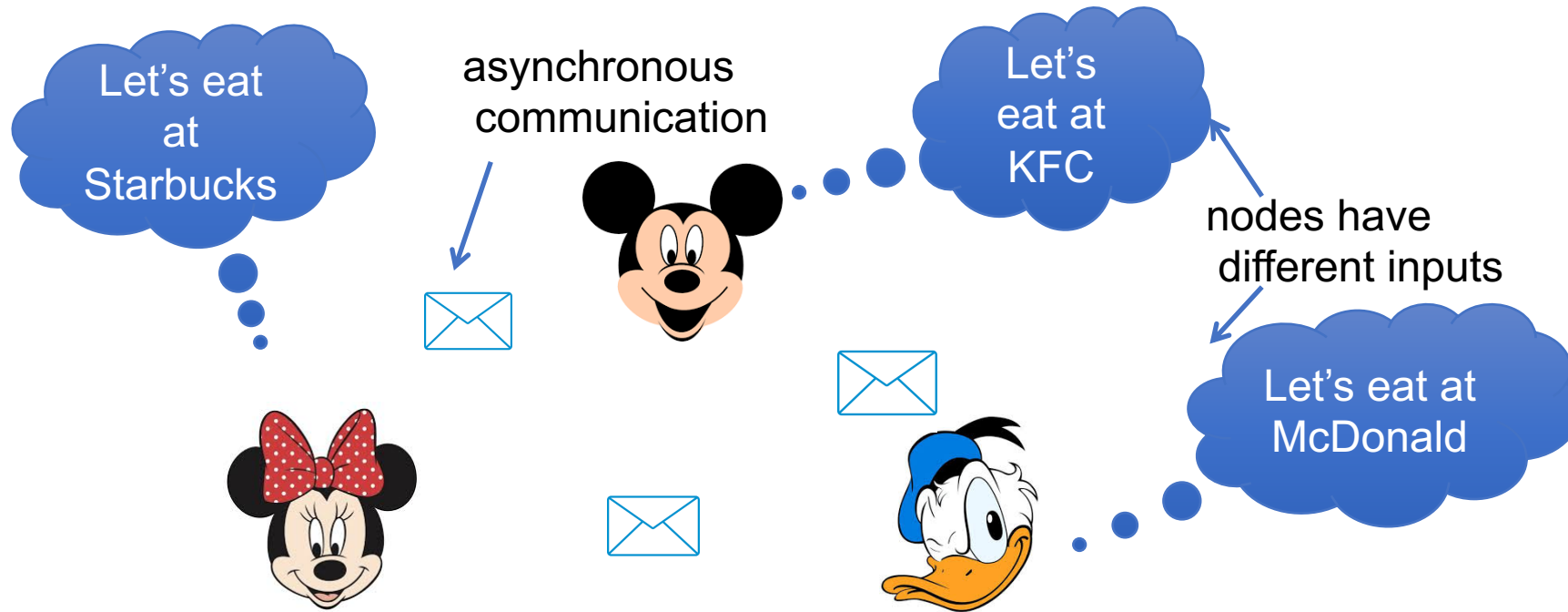
## Problem setting

- N nodes, each with a (potentially) different input
- One or more may have non-byzantine fail
- Network is asynchronous: cannot tell node crash from slow communication

## Goal

- Safety: chosen values by different nodes are the same
- Liveness: all live nodes eventually choose some value

# Single-decree consensus



# Paxos Components

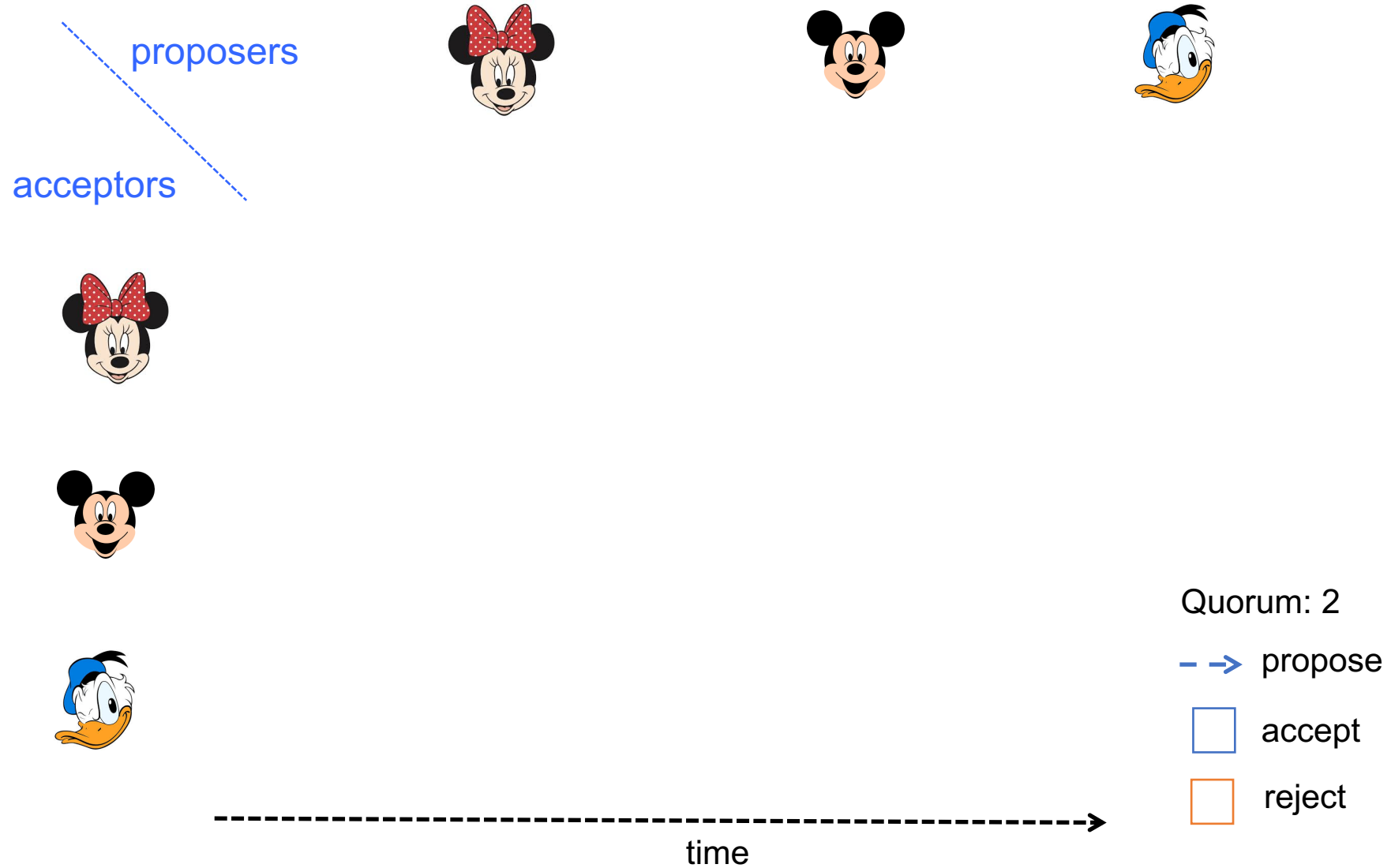
## Roles

- Proposers: send proposed values to acceptors
- Acceptors: accept/reject proposed values from proposer
- Leaners: learn about outcome of consensus (chosen value)
- For simplicity, every node plays all the roles

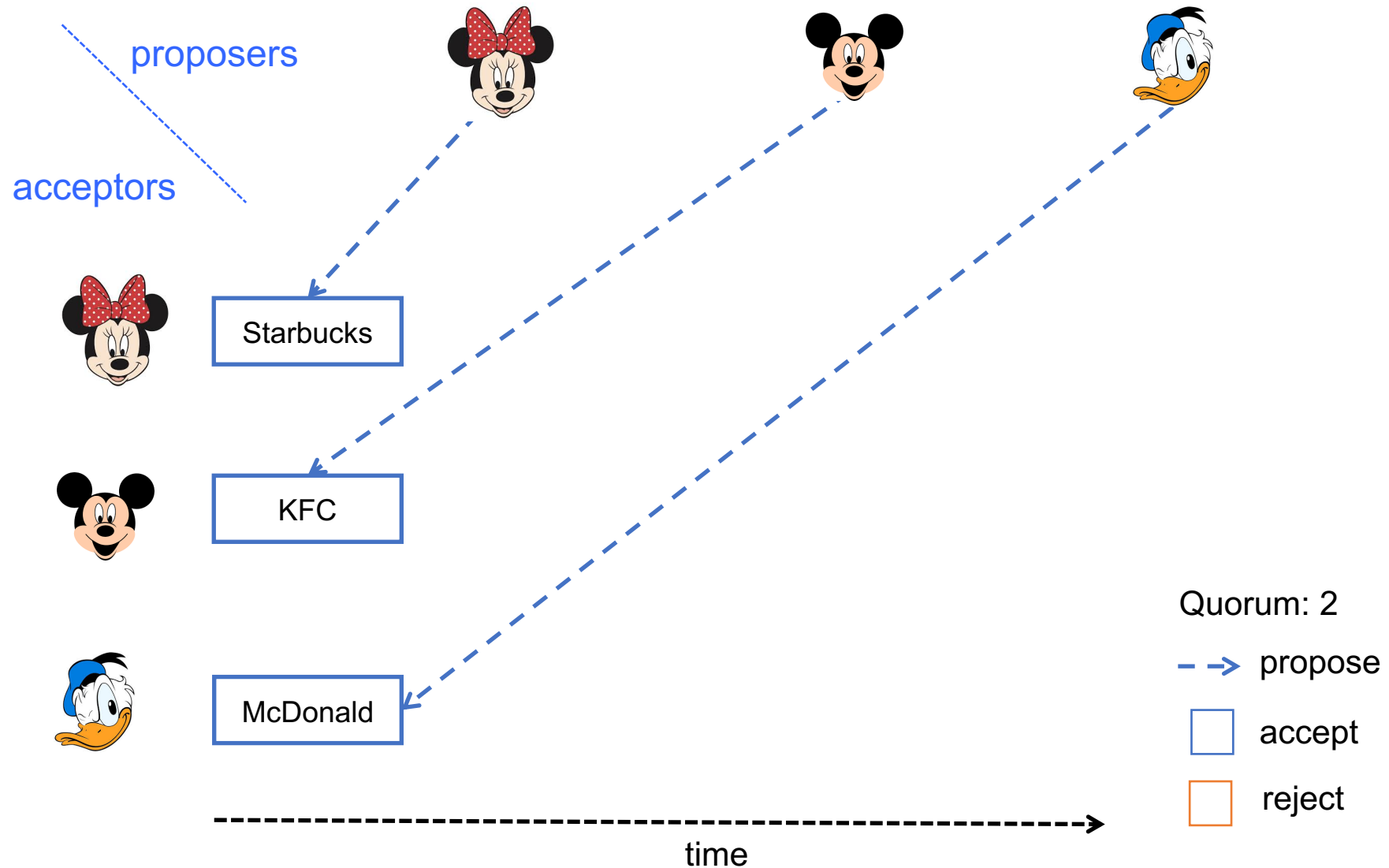
## Chosen

- A value is **chosen (decided)** if it is accepted by a quorum of nodes

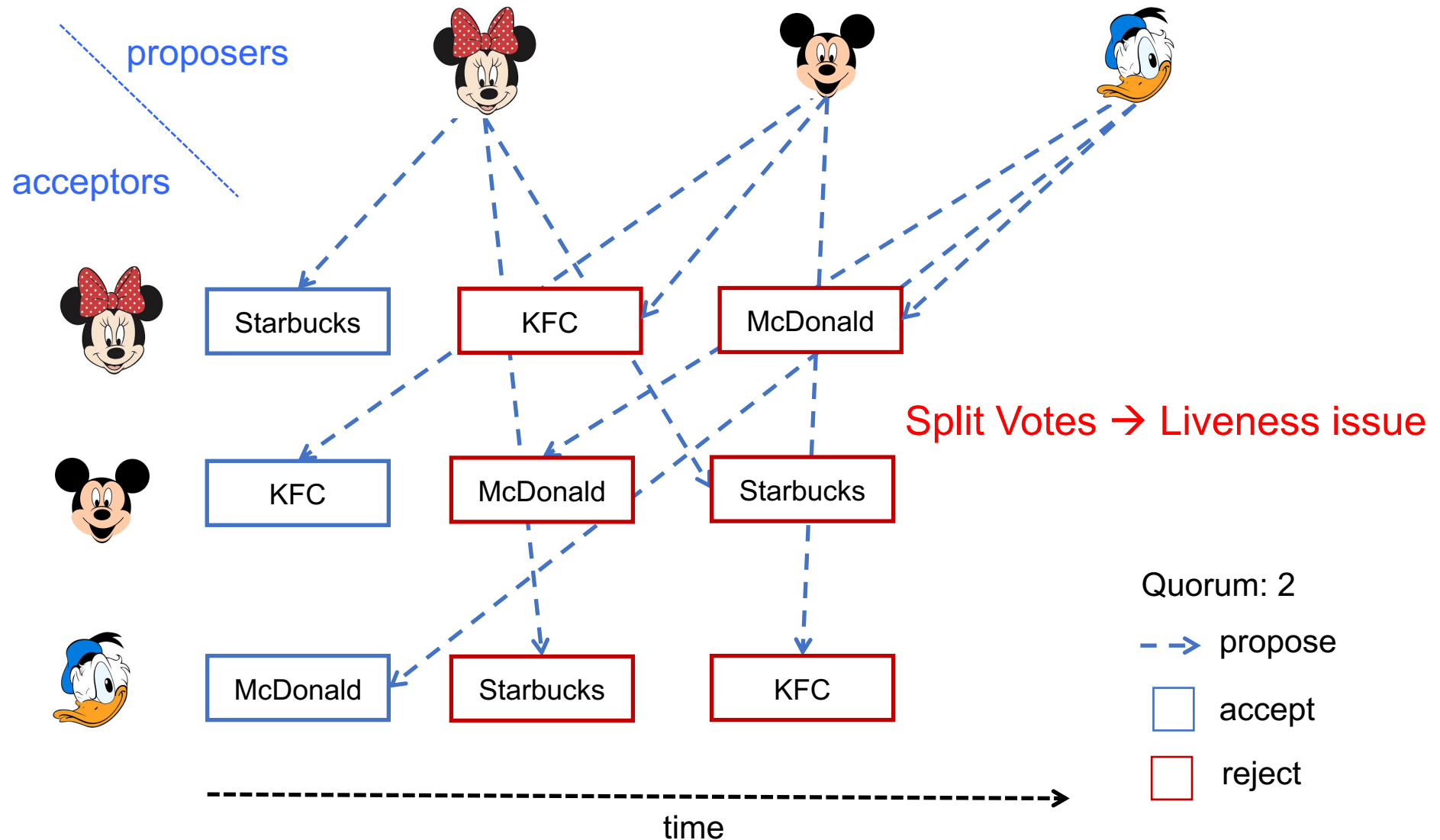
# Method I – Only accept the first



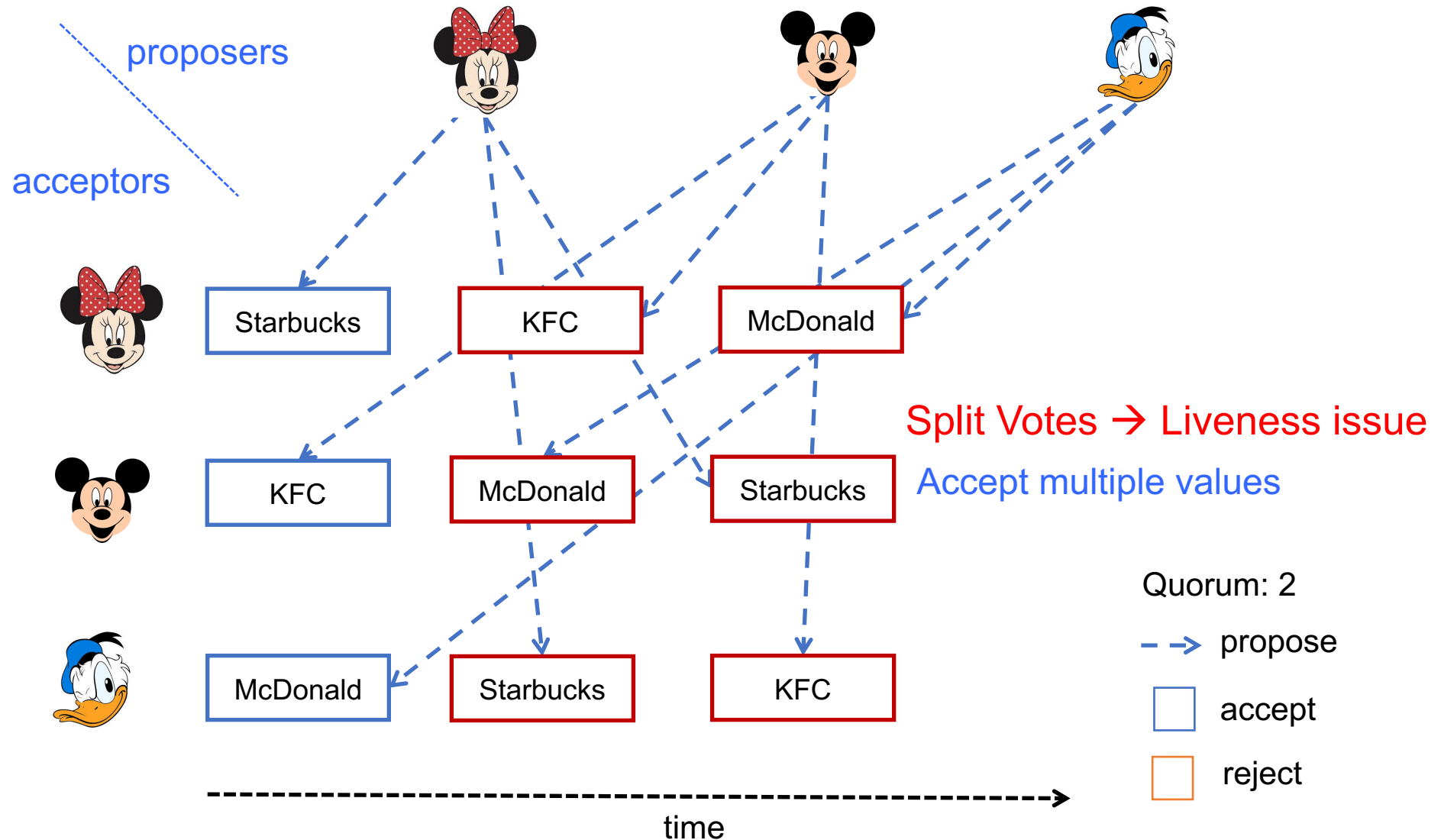
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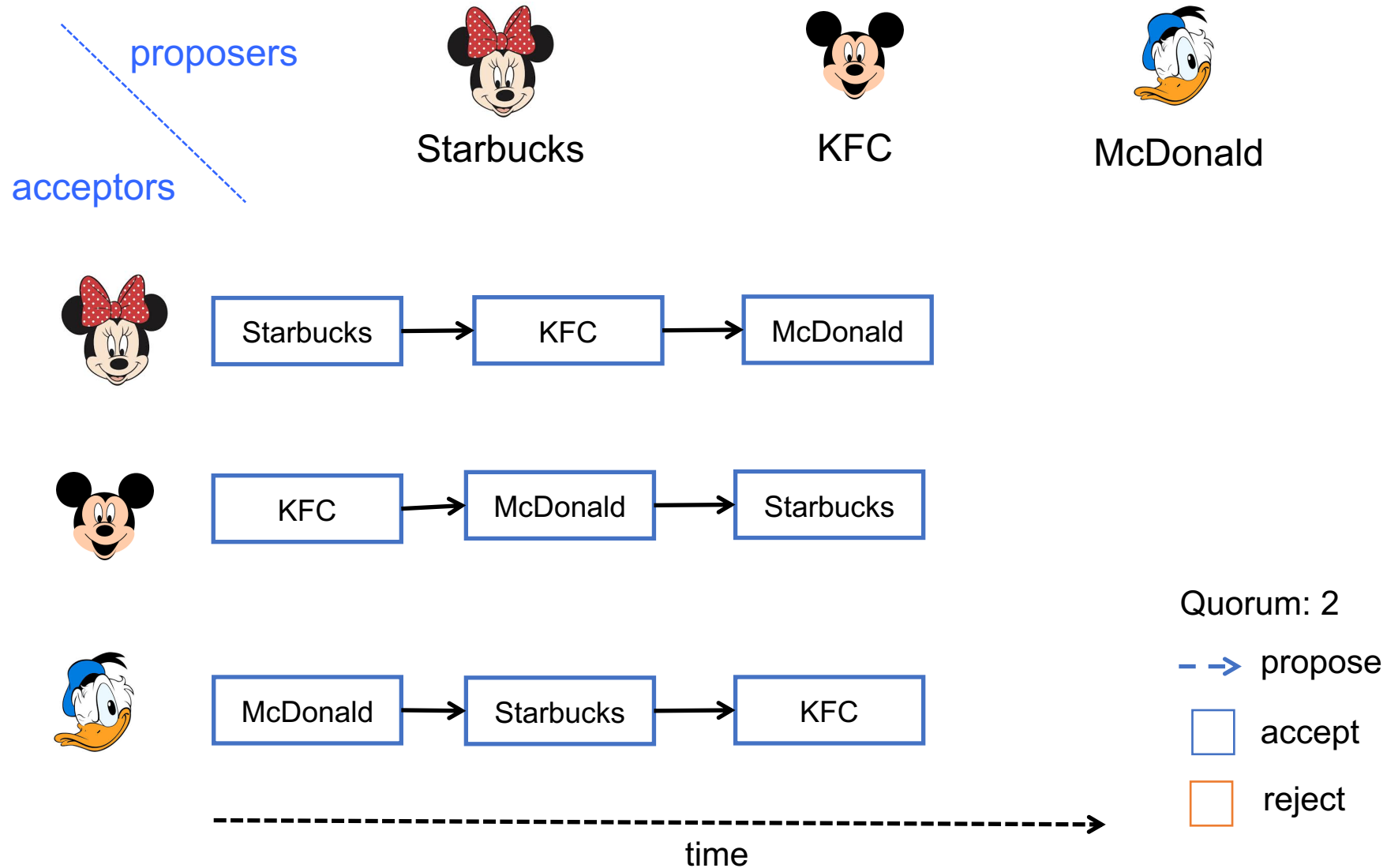
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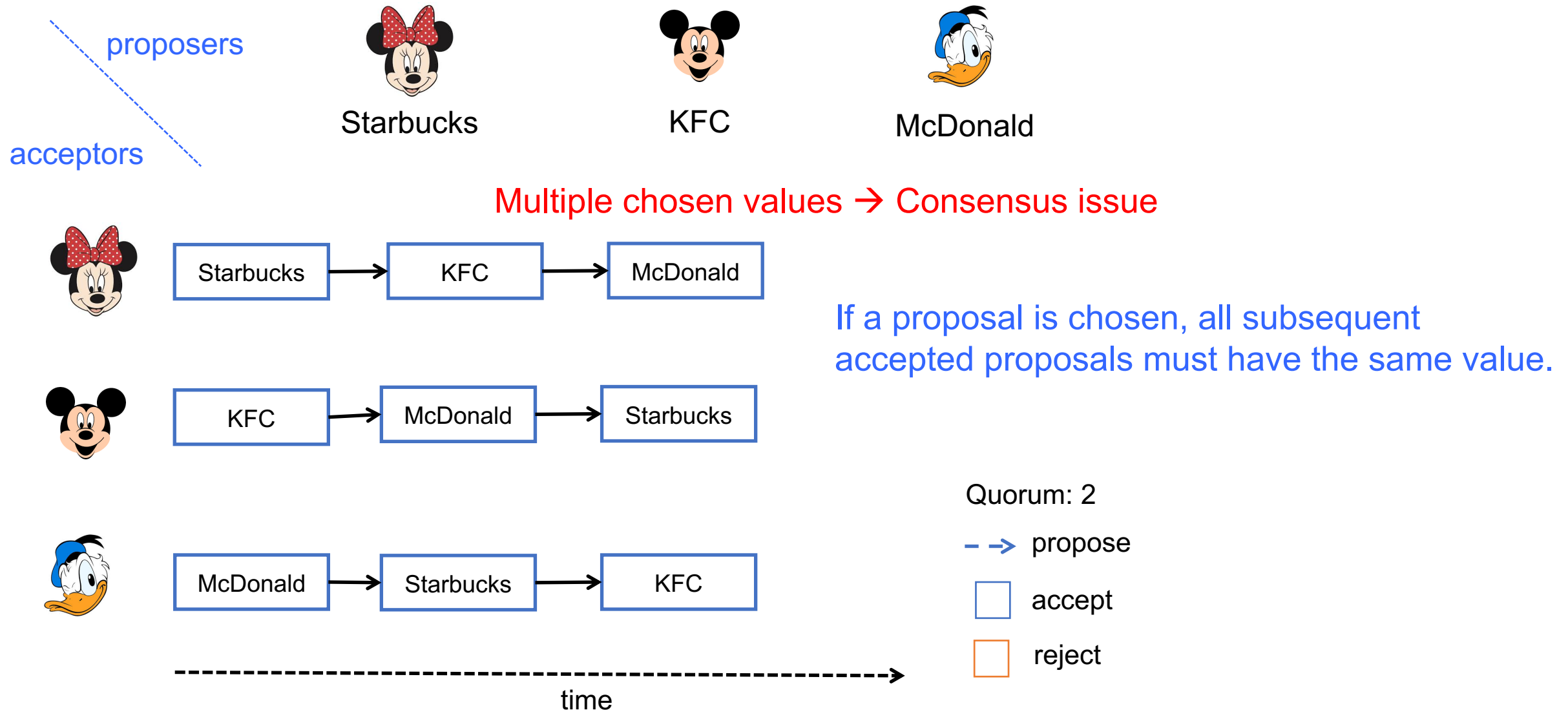
# Method I – Only accept the first



# Method II – Accept multiple proposed values



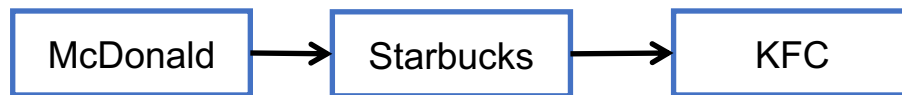
# Method II – Accept multiple proposed values



# Method II – Accept multiple proposed values



Multiple chosen values → Consensus issue



If a proposal is chosen, all subsequent accepted proposals must have the same value.  
→ Proposer should always propose the **safe value**.

Quorum: 2

— → propose

□ accept

□ reject

time →

# Basic idea of Paxos

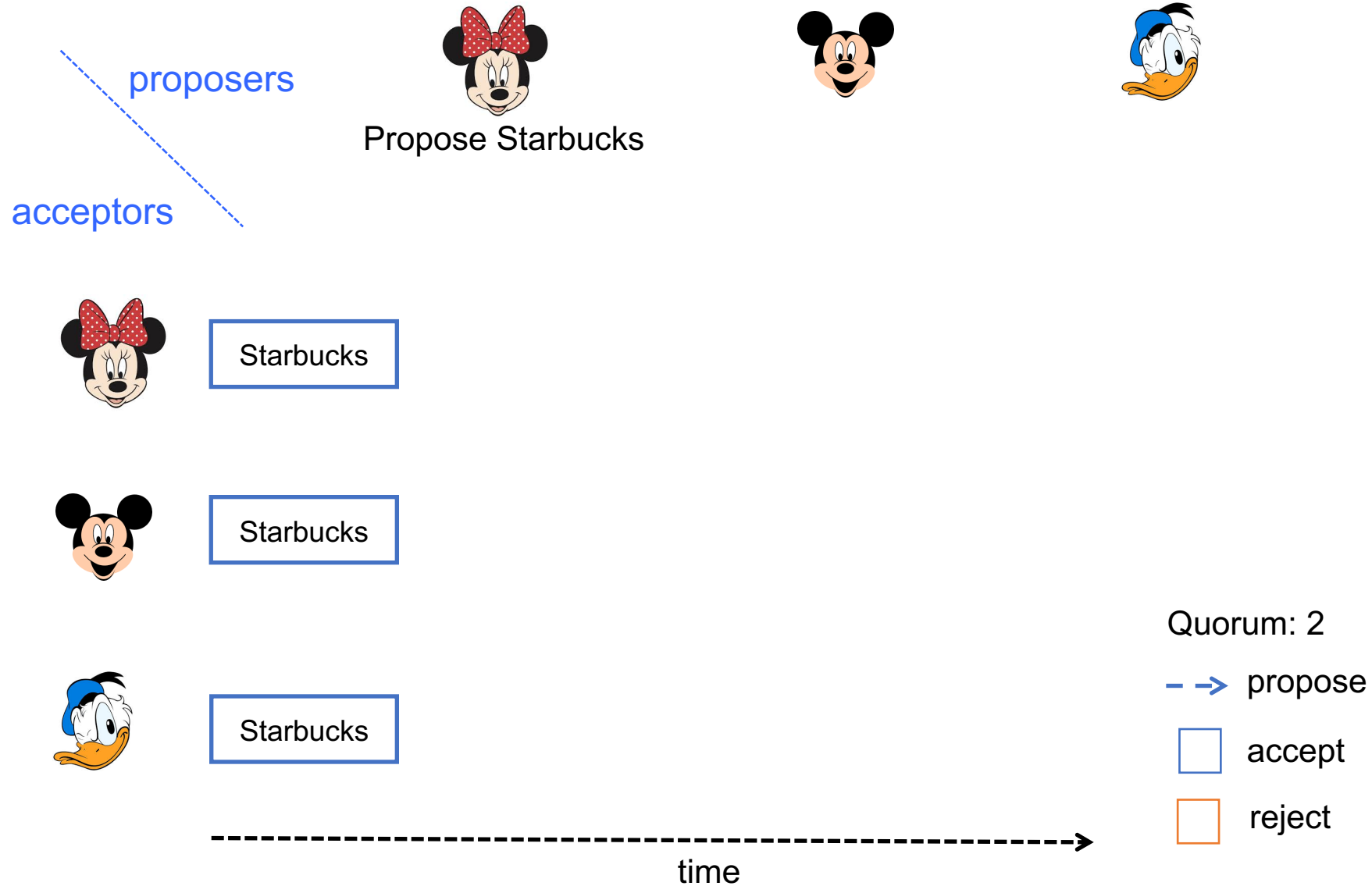
## Liveness

- Acceptors can accept more than one proposals

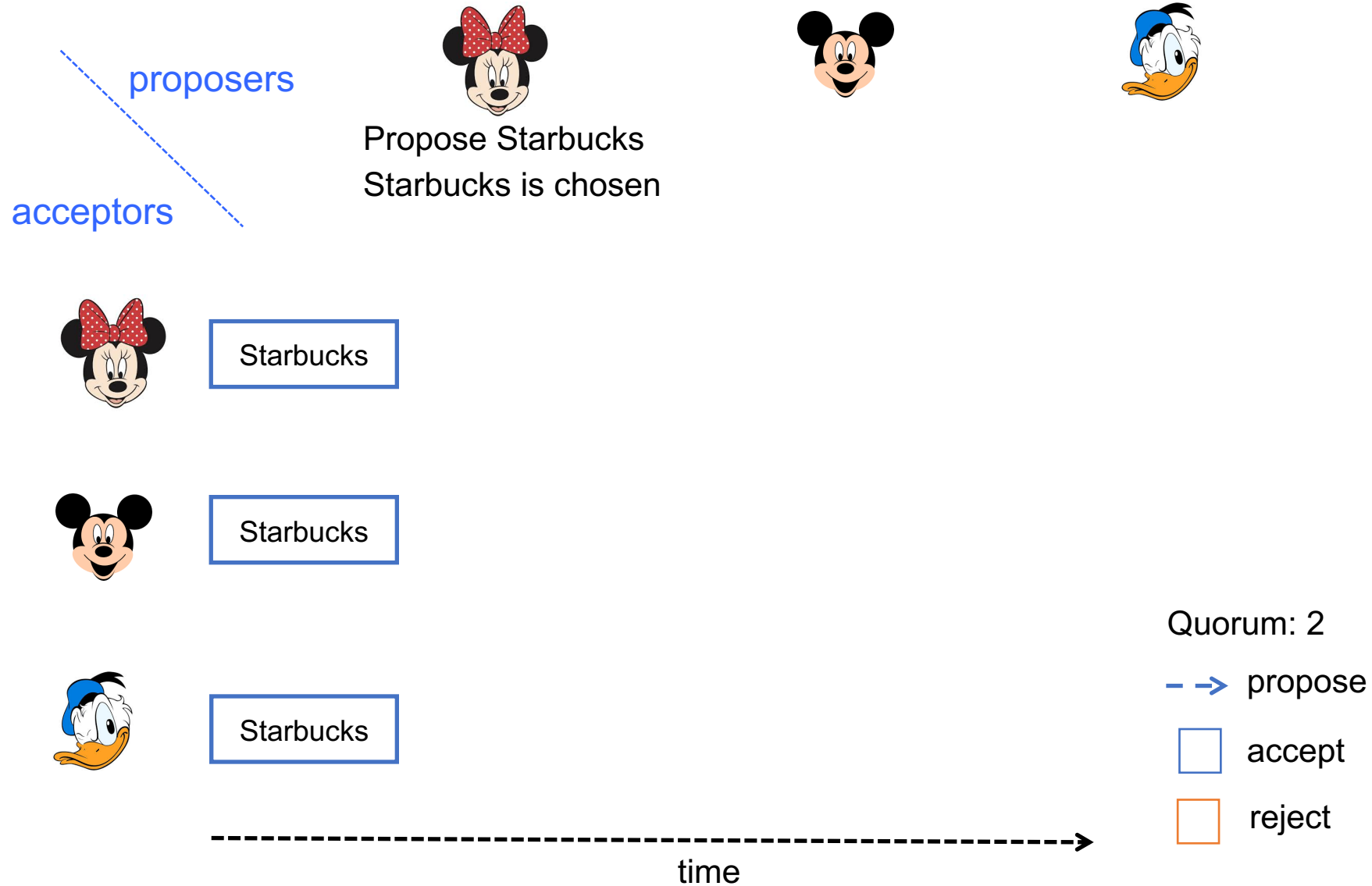
## Consensus (Safety)

- Proposers only propose safe value
  - If a proposal with value  $p$  is chosen, then  $p$  is the safe value.
  - Otherwise, any value is the safe value.

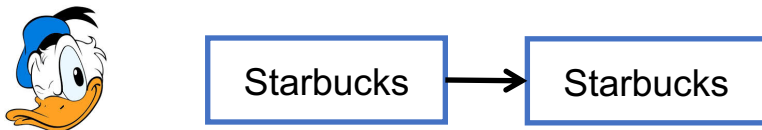
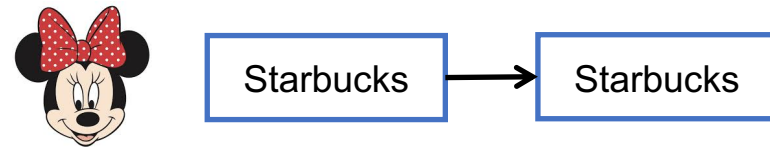
# Review The Example



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time →

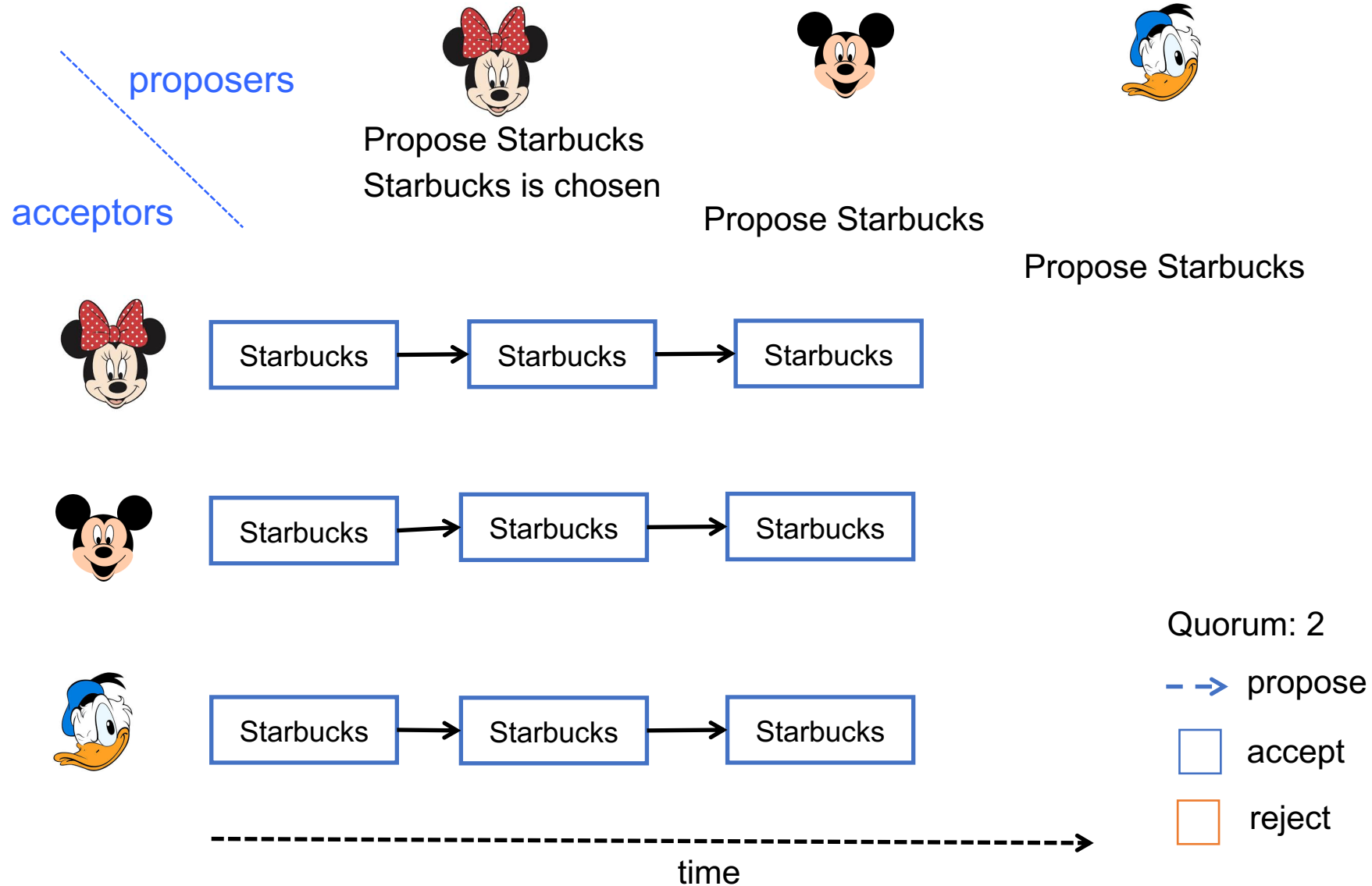
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# Review The Example



# Challenges

Discover the safe value

# Challenges

## Discover the safe value

- Use an extra round of communication to find out the safe value